

$$(c) \frac{|E_T|}{\text{True Value}} \times 100 = 0\%$$

$$\text{II. (a) For } n = 4, h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6};$$

$$\sum mf(x_i) = 36 \Rightarrow S = \frac{1}{6}(36) = 6;$$

$$f^{(4)}(x) = 0 \Rightarrow M = 0 \Rightarrow |E_S| = 0$$

$$(b) \int_1^3 (2x-1) dx = 6 \Rightarrow |E_S| = \int_1^3 (2x-1) dx - S = 6 - 6 = 0$$

$$(c) \frac{|E_S|}{\text{True Value}} \times 100 = 0\%$$

$$3. \int_{-1}^1 (x^2 + 1) dx$$

$$\text{I. (a) For } n = 4, h = \frac{b-a}{n} = \frac{1-(-1)}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{2} = \frac{1}{4};$$

$$\sum mf(x_i) = 11 \Rightarrow T = \frac{1}{4}(11) = 2.75;$$

$$f(x) = x^2 + 1 \Rightarrow f'(x) = 2x \Rightarrow f''(x) = 2 \Rightarrow M = 2$$

$$\Rightarrow |E_T| \leq \frac{1-(-1)}{12} \left(\frac{1}{2}\right)^2 (2) = \frac{1}{12} \text{ or } 0.08333$$

$$(b) \int_{-1}^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_{-1}^1 = \left(\frac{1}{3} + 1 \right) - \left(-\frac{1}{3} - 1 \right) = \frac{8}{3} \Rightarrow E_T = \int_{-1}^1 (x^2 + 1) dx - T = \frac{8}{3} - \frac{11}{4} = -\frac{1}{12}$$

$$\Rightarrow |E_T| = \left| -\frac{1}{12} \right| \approx 0.08333$$

$$(c) \frac{|E_T|}{\text{True Value}} \times 100 = \left(\frac{\frac{1}{12}}{\frac{8}{3}} \right) \times 100 \approx 3\%$$

$$\text{II. (a) For } n = 4, h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6};$$

$$\sum mf(x_i) = 16 \Rightarrow S = \frac{1}{6}(16) = \frac{8}{3} = 2.66667;$$

$$f^{(3)}(x) = 0 \Rightarrow f^{(4)}(x) = 0 \Rightarrow M = 0 \Rightarrow |E_S| = 0$$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	1	1	1	1
x_1	3/2	2	4	8
x_2	2	3	2	6
x_3	5/2	4	4	16
x_4	3	5	1	5

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	-1	2	1	2
x_1	-1/2	5/4	2	5/2
x_2	0	1	2	2
x_3	1/2	5/4	2	5/2
x_4	1	2	1	2

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	-1	2	1	2
x_1	-1/2	5/4	4	5
x_2	0	1	2	2
x_3	1/2	5/4	4	5
x_4	1	2	1	2

$$(b) \int_{-1}^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_{-1}^1 = \frac{8}{3} \Rightarrow |E_S| = \left| \int_{-1}^1 (x^2 + 1) dx - S \right| = \frac{8}{3} - \frac{8}{3} = 0$$

$$(c) \frac{|E_S|}{\text{True Value}} \times 100 = 0\%$$

$$4. \int_{-2}^0 (x^2 - 1) dx$$

$$I. (a) \text{ For } n = 4, h = \frac{b-a}{n} = \frac{0 - (-2)}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{2} = \frac{1}{4};$$

$$\sum mf(x_i) = 3 \Rightarrow T = \frac{1}{4}(3) = \frac{3}{4};$$

$$f(x) = x^2 - 1 \Rightarrow f'(x) = 2x + 1 \Rightarrow f''(x) = 2 \Rightarrow M = 2$$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	-2	3	1	3
x_1	-3/2	5/4	2	5/2
x_2	-1	0	2	0
x_3	-1/2	-3/4	2	-3/2
x_4	0	-1	1	-1

$$\Rightarrow |E_T| \leq \frac{0 - (-2)}{12} \left(\frac{1}{2} \right)^2 (2) = \frac{1}{12} = 0.08333$$

$$(b) \int_{-2}^0 (x^2 - 1) dx = \left[\frac{x^3}{3} - x \right]_{-2}^0 = 0 - \left(-\frac{8}{3} + 2 \right) = \frac{2}{3} \Rightarrow E_T = \int_{-2}^0 (x^2 - 1) dx - T = \frac{2}{3} - \frac{2}{4} = -\frac{1}{12}$$

$$\Rightarrow |E_T| = \frac{1}{12}$$

$$(c) \frac{|E_T|}{\text{True Value}} \times 100 = \left(\frac{\frac{1}{12}}{\frac{2}{3}} \right) \times 100 \approx 13\%$$

$$II. (a) \text{ For } n = 4, h = \frac{b-a}{n} = \frac{0 - (-2)}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6};$$

$$\sum mf(x_i) = 4 \Rightarrow S = \frac{1}{6}(4) = \frac{2}{3};$$

$$f^{(3)}(x) = 0 \Rightarrow f^{(4)}(x) = 0 \Rightarrow M = 0 \Rightarrow |E_S| = 0$$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	-2	3	1	3
x_1	-3/2	5/4	4	5
x_2	-1	0	2	0
x_3	-1/2	-3/4	4	-3
x_4	0	-1	1	-1

$$(b) \int_{-2}^0 (x^2 - 1) dx = \frac{2}{3} \Rightarrow |E_S| = \left| \int_{-2}^0 (x^2 - 1) dx - S \right| = \frac{2}{3} - \frac{2}{3} = 0$$

$$(c) \frac{|E_S|}{\text{True Value}} \times 100 = 0\%$$

5. $\int_0^2 (t^3 + t) dt$

I. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{2} = \frac{1}{4}$;

$$\sum mf(t_i) = 25 \Rightarrow T = \frac{1}{4}(25) = \frac{25}{4};$$

$$f(t) = t^3 + t \Rightarrow f'(t) = 3t^2 + 1 \Rightarrow f''(t) = 6t \Rightarrow M = 12$$

$$= f''(2) \Rightarrow |E_T| \leq \frac{2-0}{12} \left(\frac{1}{2}\right)^2 (12) = \frac{1}{2}$$

(b) $\int_0^2 (t^3 + t) dt = \left[\frac{t^4}{4} + \frac{t^2}{2} \right]_0^2 = \left(\frac{2^4}{4} + \frac{2^2}{2} \right) - 0 = 6 \Rightarrow |E_T| = \int_0^2 (t^3 + t) dt - T = 6 - \frac{25}{4} = -\frac{1}{4} \Rightarrow |E_T| = \frac{1}{4}$

(c) $\frac{|E_T|}{\text{True Value}} \times 100 = \frac{\left| -\frac{1}{4} \right|}{6} \times 100 \approx 4\%$

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	1/2	5/8	2	5/4
t_2	1	2	2	4
t_3	3/2	39/8	2	39/4
t_4	2	10	1	10

II. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6}$;

$$\sum mf(t_i) = 36 \Rightarrow S = \frac{1}{6}(36) = 6;$$

$$f^{(3)}(t) = 6 \Rightarrow f^{(4)}(t) = 0 \Rightarrow M = 0 \Rightarrow |E_S| = 0$$

(b) $\int_0^2 (t^3 + t) dt = 6 \Rightarrow |E_S| = \int_0^2 (t^3 + t) dt - S = 6 - 6 = 0$

(c) $\frac{|E_S|}{\text{True Value}} \times 100 = 0\%$

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	1/2	5/8	4	5/2
t_2	1	2	2	4
t_3	3/2	39/8	4	39/2
t_4	2	10	1	10

6. $\int_{-1}^1 (t^3 + 1) dt$

I. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{1-(-1)}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{2} = \frac{1}{4}$;

$$\sum mf(t_i) = 8 \Rightarrow T = \frac{1}{4}(8) = 2;$$

$$f(t) = t^3 + 1 \Rightarrow f'(t) = 3t^2 \Rightarrow f''(t) = 6t \Rightarrow M = 6$$

$$= f''(1) \Rightarrow |E_T| \leq \frac{1-(-1)}{12} \left(\frac{1}{2}\right)^2 (6) = \frac{1}{4}$$

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	-1	0	1	0
t_1	-1/2	7/8	2	7/4
t_2	0	1	2	2
t_3	1/2	9/8	2	9/4
t_4	1	2	1	2

$$(b) \int_{-1}^1 (t^3 + 1) dt = \left[\frac{t^4}{4} + t \right]_{-1}^1 = \left(\frac{1^4}{4} + 1 \right) - \left(\frac{(-1)^4}{4} + (-1) \right) = 2 \Rightarrow |E_T| = \int_{-1}^1 (t^3 + 1) dt - T = 2 - 2 = 0$$

$$(c) \frac{|E_T|}{\text{True Value}} \times 100 = 0\%$$

$$\text{II. (a) For } n = 4, h = \frac{b-a}{n} = \frac{1 - (-1)}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6};$$

$$\sum mf(t_i) = 12 \Rightarrow S = \frac{1}{6}(12) = 2;$$

$$f^{(3)}(t) = 6 \Rightarrow f^{(4)}(t) = 0 \Rightarrow M = 0 \Rightarrow |E_S| = 0$$

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	-1	0	1	0
t_1	-1/2	7/8	4	7/2
t_2	0	1	2	2
t_3	1/2	9/8	4	9/2
t_4	1	2	1	2

$$(b) \int_{-1}^1 (t^3 + 1) dt = 2 \Rightarrow |E_S| = \int_{-1}^1 (t^3 + 1) dt - S = 2 - 2 = 0$$

$$(c) \frac{|E_S|}{\text{True Value}} \times 100 = 0\%$$

$$7. \int_1^2 \frac{1}{s^2} ds$$

$$\text{I. (a) For } n = 4, h = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4} \Rightarrow \frac{h}{2} = \frac{1}{8};$$

$$\sum mf(s_i) = \frac{179,573}{44,100} \Rightarrow T = \frac{1}{8} \left(\frac{179,573}{44,100} \right) = \frac{179,573}{352,800}$$

$$\approx 0.50899; f(s) = \frac{1}{s^2} \Rightarrow f'(s) = -\frac{2}{s^3} \Rightarrow f''(s) = \frac{6}{s^4}$$

$$\Rightarrow M = 6 = f''(1) \Rightarrow |E_T| \leq \frac{2-1}{12} \left(\frac{1}{4} \right)^2 (6) = \frac{1}{32} = 0.03125$$

$$(b) \int_1^2 \frac{1}{s^2} ds = \int_1^2 s^{-2} ds = \left[-\frac{1}{s} \right]_1^2 = -\frac{1}{2} - \left(-\frac{1}{1} \right) = \frac{1}{2} \Rightarrow E_T = \int_1^2 \frac{1}{s^2} ds - T = \frac{1}{2} - 0.50899 = -0.00899$$

$$\Rightarrow |E_T| = 0.00899$$

$$(c) \frac{|E_T|}{\text{True Value}} \times 100 = \frac{0.00899}{0.5} \times 100 \approx 2\%$$

	s_i	$f(s_i)$	m	$mf(s_i)$
s_0	1	1	1	1
s_1	5/4	16/25	2	32/25
s_2	3/2	4/9	2	8/9
s_3	7/4	16/49	2	32/49
s_4	2	1/4	1	1/4

$$\text{II. (a) For } n = 4, h = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4} \Rightarrow \frac{h}{3} = \frac{1}{12};$$

$$\sum mf(s_i) = \frac{264,821}{44,100} \Rightarrow S = \frac{1}{12} \left(\frac{264,821}{44,100} \right) = \frac{264,821}{529,200}$$

$$\approx 0.50042; f^{(3)}(s) = -\frac{24}{s^5} \Rightarrow f^{(4)}(s) = \frac{120}{s^6}$$

	s_i	$f(s_i)$	m	$mf(s_i)$
s_0	1	1	1	1
s_1	5/4	16/25	4	64/25
s_2	3/2	4/9	2	8/9
s_3	7/4	16/49	4	64/49
s_4	2	1/4	1	1/4

$$\Rightarrow M = 120 = f^{(4)}(1) \Rightarrow |E_S| \leq \frac{2-1}{180} \left(\frac{1}{4}\right)^4 (120) = \frac{1}{384} \approx 0.002604$$

$$(b) \int_1^2 \frac{1}{s^2} ds = \frac{1}{2} \Rightarrow E_S = \int_1^2 \frac{1}{s^2} ds - S = \frac{1}{2} - 0.050041 = -0.00041 \Rightarrow |E_S| = 0.00041$$

$$(c) \frac{|E_S|}{\text{True Value}} \times 100 = \frac{0.0004}{0.5} \times 100 \approx 0.08\%$$

$$8. \int_2^4 \frac{1}{(s-1)^2} ds$$

$$I. (a) \text{ For } n = 4, h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2} \Rightarrow \frac{h}{2} = \frac{1}{4};$$

$$\sum mf(s_i) = \frac{1269}{450} \Rightarrow T = \frac{1}{4} \left(\frac{1269}{450} \right) = \frac{1269}{1800} = 0.70500;$$

$$f(s) = (s-1)^{-2} \Rightarrow f'(s) = -\frac{2}{(s-1)^3}$$

$$\Rightarrow f''(s) = \frac{6}{(s-1)^4} \Rightarrow M = 6 = f''(2)$$

$$\Rightarrow |E_T| \leq \frac{4-2}{12} \left(\frac{1}{2}\right)^2 (6) = \frac{1}{4} = 0.25$$

$$(b) \int_2^4 \frac{1}{(s-1)^2} ds = \left[\frac{-1}{(s-1)} \right]_2^4 = \left(\frac{-1}{4-1} \right) - \left(\frac{-1}{2-1} \right) = \frac{2}{3} \Rightarrow E_T = \int_2^4 \frac{1}{(s-1)^2} ds - T = \frac{2}{3} - 0.705 \approx -0.03833$$

$$\Rightarrow |E_T| \approx 0.03833$$

$$(c) \frac{|E_T|}{\text{True Value}} \times 100 = \frac{0.03833}{\left(\frac{2}{3}\right)} \times 100 \approx 6\%$$

	s_i	$f(s_i)$	m	$mf(s_i)$
s_0	2	1	1	1
s_1	5/2	4/9	2	8/9
s_2	3	1/4	2	1/2
s_3	7/2	4/25	2	8/25
s_4	4	1/9	1	1/9

$$II. (a) \text{ For } n = 4, h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6};$$

$$\sum mf(s_i) = \frac{1813}{450} \Rightarrow S = \frac{1}{6} \left(\frac{1813}{450} \right) = \frac{1813}{2700} \approx 0.67148;$$

$$f^{(3)}(s) = \frac{-24}{(s-1)^5} \Rightarrow f^{(4)}(s) = \frac{120}{(s-1)^6} \Rightarrow M = 120$$

$$= f^{(4)}(2) \Rightarrow |E_S| \leq \frac{4-2}{180} \left(\frac{1}{2}\right)^4 (120) = \frac{1}{12} \approx 0.08333$$

$$(b) \int_2^4 \frac{1}{(s-1)^2} ds = \frac{2}{3} \Rightarrow E_S = \int_2^4 \frac{1}{(s-1)^2} ds - S \approx \frac{2}{3} - 0.67148 = -0.00481 \Rightarrow |E_S| \approx 0.00481$$

$$(c) \frac{|E_S|}{\text{True Value}} \times 100 = \frac{0.00481}{\left(\frac{2}{3}\right)} \times 100 \approx 1\%$$

	s_i	$f(s_i)$	m	$mf(s_i)$
s_0	2	1	1	1
s_1	5/2	4/9	4	16/9
s_2	3	1/4	2	1/2
s_3	7/2	4/25	4	16/25
s_4	4	1/9	1	1/9

9. $\int_0^{\pi} \sin t \, dt$

I. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{\pi-0}{4} = \frac{\pi}{4} \Rightarrow \frac{h}{2} = \frac{\pi}{8}$;

$$\sum mf(t_i) = 2 + 2\sqrt{2} \approx 4.8284 \Rightarrow T = \frac{\pi}{8}(2 + 2\sqrt{2})$$

$$\approx 1.89612; f(t) = \sin t \Rightarrow f'(t) = \cos t \Rightarrow f''(t) = -\sin t$$

$$\Rightarrow M = 1 = |f''(0)| \Rightarrow |E_T| \leq \frac{\pi-0}{12} \left(\frac{\pi}{4}\right)^2 (1) = \frac{\pi^3}{192} \approx 0.16149$$

(b) $\int_0^{\pi} \sin t \, dt = [-\cos t]_0^{\pi} = (-\cos \pi) - (-\cos 0) = 2 \Rightarrow |E_T| = \left| \int_0^{\pi} \sin t \, dt - T \right| \approx 2 - 1.89612 = 0.10388$

(c) $\frac{|E_T|}{\text{True Value}} \times 100 = \frac{0.10388}{2} \times 100 \approx 5\%$

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	$\pi/4$	$\sqrt{2}/2$	2	$\sqrt{2}$
t_2	$\pi/2$	1	2	2
t_3	$3\pi/4$	$\sqrt{2}/2$	2	$\sqrt{2}$
t_4	π	0	1	0

II. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{\pi-0}{4} = \frac{\pi}{4} \Rightarrow \frac{h}{3} = \frac{\pi}{12}$;

$$\sum mf(t_i) = 2 + 4\sqrt{2} \approx 7.6569 \Rightarrow S = \frac{\pi}{12}(2 + 4\sqrt{2})$$

$$\approx 2.00456; f^{(3)}(t) = -\cos t \Rightarrow f^{(4)}(t) = \sin t$$

$$\Rightarrow M = 1 = f^{(4)}(0) \Rightarrow |E_S| \leq \frac{\pi-0}{180} \left(\frac{\pi}{4}\right)^4 (1) \approx 0.00664$$

(b) $\int_0^{\pi} \sin t \, dt = 2 \Rightarrow E_S = \int_0^{\pi} \sin t \, dt - S \approx 2 - 2.00456 = -0.00456 \Rightarrow |E_S| \approx 0.00456$

(c) $\frac{|E_S|}{\text{True Value}} \times 100 = \frac{0.00456}{2} \times 100 \approx 0.23\%$

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	$\pi/4$	$\sqrt{2}/2$	4	$2\sqrt{2}$
t_2	$\pi/2$	1	2	2
t_3	$3\pi/4$	$\sqrt{2}/2$	4	$2\sqrt{2}$
t_4	π	0	1	0

10. $\int_0^1 \sin \pi t \, dt$

I. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} \Rightarrow \frac{h}{2} = \frac{1}{8}$;

$$\sum mf(t_i) = 2 + 2\sqrt{2} \approx 4.828 \Rightarrow T = \frac{1}{8}(2 + 2\sqrt{2})$$

$$\approx 0.60355; f(t) = \sin \pi t \Rightarrow f'(t) = \pi \cos \pi t$$

$$\Rightarrow f''(t) = -\pi^2 \sin \pi t \Rightarrow M = \pi^2 = |f''(0)|$$

$$\Rightarrow |E_T| \leq \frac{1-0}{12} \left(\frac{1}{4}\right)^2 (\pi^2) \approx 0.05140$$

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	$1/4$	$\sqrt{2}/2$	2	$\sqrt{2}$
t_2	$1/2$	1	2	2
t_3	$3/4$	$\sqrt{2}/2$	2	$\sqrt{2}$
t_4	1	0	1	0

$$(b) \int_0^1 \sin \pi t \, dt = \left[-\frac{1}{\pi} \cos \pi t \right]_0^1 = \left(-\frac{1}{\pi} \cos \pi \right) - \left(-\frac{1}{\pi} \cos 0 \right) = \frac{2}{\pi} \approx 0.63662 \Rightarrow |E_T| = \int_0^1 \sin \pi t \, dt - T \\ \approx \frac{2}{\pi} - 0.60355 = 0.03307$$

$$(c) \frac{|E_T|}{\text{True Value}} \times 100 = \frac{0.03307}{\left(\frac{2}{\pi}\right)} \times 100 \approx 5\%$$

II. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} \Rightarrow \frac{h}{3} = \frac{1}{12}$;

$$\sum mf(t_i) = 2 + 4\sqrt{2} \approx 7.65685 \Rightarrow S = \frac{1}{12}(2 + 4\sqrt{2})$$

$$\approx 0.63807; f^{(3)}(t) = -\pi^3 \cos \pi t \Rightarrow f^{(4)}(t) = \pi^4 \sin \pi t$$

$$\Rightarrow M = \pi^4 = f^{(4)}(0) \Rightarrow |E_S| \leq \frac{1-0}{180} \left(\frac{1}{4}\right)^4 (\pi^4) \approx 0.00211$$

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	1/4	$\sqrt{2}/2$	4	$2\sqrt{2}$
t_2	1/2	1	2	2
t_3	3/4	$\sqrt{2}/2$	4	$2\sqrt{2}$
t_4	1	0	1	0

$$(b) \int_0^1 \sin \pi t \, dt = \frac{2}{\pi} \approx 0.63662 \Rightarrow E_S = \int_0^1 \sin \pi t \, dt - S \approx \frac{2}{\pi} - 0.63807 = -0.00145 \Rightarrow |E_S| \approx 0.00145$$

$$(c) \frac{|E_S|}{\text{True Value}} \times 100 = \frac{0.00145}{\left(\frac{2}{\pi}\right)} \times 100 \approx 0\%$$

11. (a) $n = 8 \Rightarrow h = \frac{1}{8} \Rightarrow \frac{h}{2} = \frac{1}{16}$;

$$\sum mf(x_i) = 1(0.0) + 2(0.12402) + 2(0.24206) + 2(0.34763) + 2(0.43301) + 2(0.48789) + 2(0.49608) \\ + 2(0.42361) + 1(0) = 5.1086 \Rightarrow T = \frac{1}{16}(5.1086) = 0.31929$$

(b) $n = 8 \Rightarrow h = \frac{1}{8} \Rightarrow \frac{h}{3} = \frac{1}{24}$;

$$\sum mf(x_i) = 1(0.0) + 4(0.12402) + 2(0.24206) + 4(0.34763) + 2(0.43301) + 4(0.48789) + 2(0.49608) \\ + 4(0.42361) + 1(0) = 7.8749 \Rightarrow S = \frac{1}{24}(7.8749) = 0.32812$$

(c) Let $u = 1 - x^2 \Rightarrow du = -2x \, dx \Rightarrow -\frac{1}{2} du = x \, dx$; $x = 0 \Rightarrow u = 1$, $x = 1 \Rightarrow u = 0$

$$\int_0^1 x\sqrt{1-x^2} \, dx = \int_1^0 \sqrt{u} \left(-\frac{1}{2} du\right) = \frac{1}{2} \int_0^1 u^{1/2} \, du = \left[\frac{1}{2} \left(\frac{u^{3/2}}{3/2} \right) \right]_0^1 = \left[\frac{1}{3} u^{3/2} \right]_0^1 = \frac{1}{3}(\sqrt{1})^3 - \frac{1}{3}(\sqrt{0})^3 = \frac{1}{3};$$

$$E_T = \int_0^1 x\sqrt{1-x^2} \, dx - T \approx \frac{1}{3} - 0.31929 = 0.01404; E_S = \int_0^1 x\sqrt{1-x^2} \, dx - S \approx \frac{1}{3} - 0.32812 = 0.00521$$

12. (a) $n = 8 \Rightarrow h = \frac{3}{8} \Rightarrow \frac{h}{2} = \frac{3}{16}$;

$$\sum mf(\theta_i) = 1(0) + 2(0.09334) + 2(0.18429) + 2(0.27075) + 2(0.35112) + 2(0.42443) + 2(0.49026) \\ + 2(0.58466) + 1(0.6) = 5.3977 \Rightarrow T = \frac{3}{16}(5.3977) = 1.01207$$

$$(b) \ n = 8 \Rightarrow h = \frac{3}{8} \Rightarrow \frac{h}{3} = \frac{1}{8};$$

$$\sum mf(\theta_i) = 1(0) + 4(0.09334) + 2(0.18429) + 4(0.27075) + 2(0.35112) + 4(0.42443) + 2(0.49026) \\ + 4(0.58466) + 1(0.6) = 8.14406 \Rightarrow S = \frac{1}{8}(8.14406) = 1.01801$$

$$(c) \text{ Let } u = 16 + \theta^2 \Rightarrow du = 2\theta \, d\theta \Rightarrow \frac{1}{2} du = \theta \, d\theta; \theta = 0 \Rightarrow u = 16, \theta = 3 \Rightarrow u = 16 + 3^2 = 25$$

$$\int_0^3 \frac{\theta}{\sqrt{16 + \theta^2}} \, d\theta = \int_{16}^{25} \frac{1}{\sqrt{u}} \left(\frac{1}{2} du\right) = \frac{1}{2} \int_{16}^{25} u^{-1/2} \, du = \left[\frac{1}{2} \left(\frac{u^{1/2}}{1/2} \right) \right]_{16}^{25} = \sqrt{25} - \sqrt{16} = 1;$$

$$E_T = \int_0^3 \frac{\theta}{\sqrt{16 + \theta^2}} \, d\theta - T \approx 1 - 1.01207 = -0.01207; E_S = \int_0^3 \frac{\theta}{\sqrt{16 + \theta^2}} \, d\theta - S \approx 1 - 0.01801 = -0.01801$$

$$13. (a) \ n = 8 \Rightarrow h = \frac{\pi}{8} \Rightarrow \frac{h}{2} = \frac{\pi}{16};$$

$$\sum mf(t_i) = 1(0.0) + 2(0.99138) + 2(1.26906) + 2(1.05961) + 2(0.75) + 2(0.48821) + 2(0.28946) + 2(0.13429) \\ + 1(0) = 9.96402 \Rightarrow T = \frac{\pi}{16}(9.96402) \approx 1.95643$$

$$(b) \ n = 8 \Rightarrow h = \frac{\pi}{8} \Rightarrow \frac{h}{3} = \frac{\pi}{24};$$

$$\sum mf(t_i) = 1(0.0) + 4(0.99138) + 2(1.26906) + 4(1.05961) + 2(0.75) + 4(0.48821) + 2(0.28946) + 4(0.13429) \\ + 1(0) = 15.311 \Rightarrow S \approx \frac{\pi}{24}(15.311) \approx 2.00421$$

$$(c) \text{ Let } u = 2 + \sin t \Rightarrow du = \cos t \, dt; t = -\frac{\pi}{2} \Rightarrow u = 2 + \sin\left(-\frac{\pi}{2}\right) = 1, t = \frac{\pi}{2} \Rightarrow u = 2 + \sin \frac{\pi}{2} = 3$$

$$\int_{-\pi/2}^{\pi/2} \frac{3 \cos t}{(2 + \sin t)^2} \, dt = \int_1^3 \frac{3}{u^2} \, du = 3 \int_1^3 u^{-2} \, du = \left[3 \left(\frac{u^{-1}}{-1} \right) \right]_1^3 = 3 \left(-\frac{1}{3} \right) - 3 \left(-\frac{1}{1} \right) = 2;$$

$$E_T = \int_{-\pi/2}^{\pi/2} \frac{3 \cos t}{(2 + \sin t)^2} \, dt - T \approx 2 - 1.95643 = 0.04357; E_S = \int_{-\pi/2}^{\pi/2} \frac{3 \cos t}{(2 + \sin t)^2} \, dt - S \\ \approx 2 - 2.00421 = -0.00421$$

$$14. (a) \ n = 8 \Rightarrow h = \frac{\pi}{32} \Rightarrow \frac{h}{2} = \frac{\pi}{64};$$

$$\sum mf(y_i) = 1(2.0) + 2(1.51606) + 2(1.18237) + 2(0.93998) + 2(0.75402) + 2(0.60145) + 2(0.46364) \\ + 2(0.31688) + 1(0) = 13.5488 \Rightarrow T \approx \frac{\pi}{64}(13.5488) = 0.66508$$

$$(b) \ n = 8 \Rightarrow h = \frac{\pi}{32} \Rightarrow \frac{h}{3} = \frac{\pi}{96};$$

$$\sum mf(y_i) = 1(2.0) + 4(1.51606) + 2(1.18237) + 4(0.93998) + 2(0.75402) + 4(0.60145) + 2(0.46364) \\ + 4(0.31688) + 1(0) = 20.29754 \Rightarrow S \approx \frac{\pi}{96}(20.29754) = 0.66424$$

$$(c) \text{ Let } u = \cot y \Rightarrow du = -\csc^2 y \, dy; y = \frac{\pi}{4} \Rightarrow u = 1, y = \frac{\pi}{2} \Rightarrow u = 0$$

$$\int_{\pi/4}^{\pi/2} (\csc^2 y) \sqrt{\cot y} \, dy = \int_1^0 \sqrt{u} \, (-du) = \int_0^1 u^{1/2} \, du = \left[\frac{u^{3/2}}{3/2} \right]_0^1 = \frac{2}{3} (\sqrt{1})^3 - \frac{2}{3} (\sqrt{0})^3 = \frac{2}{3};$$

$$E_T = \int_{\pi/4}^{\pi/2} (\csc^2 y) \sqrt{\cot y} \, dy - T \approx \frac{2}{3} - 0.66508 = 0.00159; \quad E_S = \int_{\pi/4}^{\pi/2} (\csc^2 y) \sqrt{\cot y} \, dy - S \\ \approx \frac{2}{3} - 0.66424 = 0.00243$$

$$15. \frac{5}{2}(6.0 + 2(8.2) + 2(9.1 + \dots + 2(12.7) + 13.0)(30) = 15,990 \text{ ft}^3$$

$$16. (a) \text{ Using the Trapezoid Rule, } h = 200 \Rightarrow \frac{h}{2} = \frac{200}{2} = 100;$$

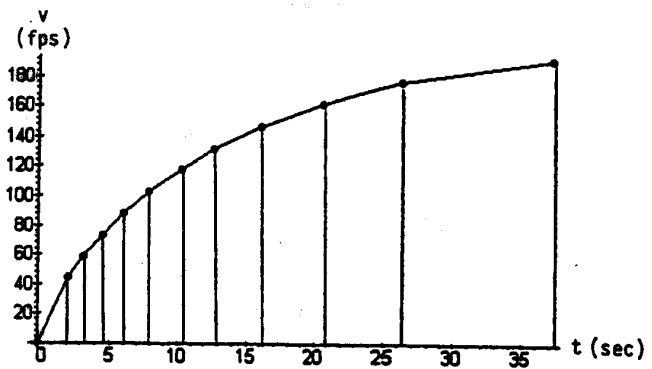
$$\sum mf(x_i) = 13,180 \Rightarrow \text{Area} \approx 100(13,180)$$

$= 1,318,000 \text{ ft}^2$. Since the average depth $= 20 \text{ ft}$
we obtain $\text{Volume} \approx 20 (\text{Area}) \approx 26,360,000 \text{ ft}^3$.

$$(b) \text{ The number of fish} = \frac{\text{Volume}}{1000} = 26,360 \text{ (to the nearest fish)} \Rightarrow \text{Maximum to be caught} = 75\% \text{ of } 26,360 = 19,770 \\ \Rightarrow \text{Number of licenses} = \frac{19,770}{20} = 988$$

	x_i	$f(x_i)$	m	$mf(x_i)$
	x_0	0	1	0
	x_1	200	2	1040
	x_2	400	2	1600
	x_3	600	2	2000
	x_4	800	2	2280
	x_4	1000	2	2320
	x_6	1200	2	2220
	x_7	1400	2	1720
	x_8	1600	1	0

17. Use the conversion $30 \text{ mph} = 44 \text{ fps}$ (ft per sec) since time is measured in seconds. The distance traveled as the car accelerates from, say, $40 \text{ mph} = 58.67 \text{ fps}$ to $50 \text{ mph} = 73.33 \text{ fps}$ in $(4.5 - 3.2) = 1.3 \text{ sec}$ is the area of the trapezoid (see figure) associated with that time interval: $\frac{1}{2}(58.67 + 73.33)(1.3) = 85.8 \text{ ft}$. The total distance traveled by the Ford Mustang Cobra is the sum of all these eleven trapezoids (using $\frac{\Delta t}{2}$ and the table below):



$$s = (44)(1.1) + (102.67)(0.5) + (132)(0.65) + (161.33)(0.7) + (190.67)(0.95) + (220)(1.2) + (249.33)(1.25) \\ + (278.67)(1.65) + (308)(2.3) + (337.33)(2.8) + (366.67)(5.45) = 5166.33 \text{ ft} \approx 0.978 \text{ mi}$$

v (mph)	0	30	40	50	60	70	80	90	100	110	120	130
v (fps)	0	44	58.67	73.33	88	102.67	117.33	132	146.67	161.33	176	190.67
t (sec)	0	2.2	3.2	4.5	5.9	7.8	10.2	12.7	16	20.6	26.2	37.1
$\Delta t/2$	0	1.1	0.5	0.65	0.7	0.95	1.2	1.25	1.65	2.3	2.8	5.45

18. Using Simpson's Rule, $h = \frac{b-a}{n} = \frac{24-0}{6} = \frac{24}{6} = 4$;

$$\sum my_i = 350 \Rightarrow S = \frac{4}{3}(350) = \frac{1400}{3} \approx 466.7 \text{ in.}^2$$

	x_i	y_i	m	my_i
x_0	0	0	1	0
x_1	4	18.75	4	75
x_2	8	24	2	48
x_3	12	26	4	104
x_4	16	24	2	48
x_5	20	18.75	4	75
x_6	24	0	1	0

19. Using Simpson's Rule, $h = 1 \Rightarrow \frac{h}{3} = \frac{1}{3}$;

$$\sum my_i = 33.6 \Rightarrow \text{Cross Section Area} \approx \frac{1}{3}(33.6) = 11.2 \text{ ft}^2.$$

Let x be the length of the tank. Then the Volume V

= (Cross Sectional Area) $x = 11.2x$. Now 5000 lb of

gasoline at 42 lb/ft³ $\Rightarrow V = \frac{5000}{42} = 119.05 \text{ ft}^3$

$$\Rightarrow 119.05 = 11.2x \Rightarrow x \approx 10.63 \text{ ft}$$

	x_i	y_i	m	my_i
x_0	0	1.5	1	1.5
x_1	1	1.6	4	6.4
x_2	2	1.8	2	3.6
x_3	3	1.9	4	7.6
x_4	4	2.0	2	4.0
x_5	5	2.1	4	8.4
x_6	6	2.1	1	2.1

20. Using Simpson's Rule, $h = 24 \Rightarrow \frac{h}{2} = 12$; $12[0.019 + 2(0.020) + 2(0.021) + \dots + 2(0.031) + 0.035] = 4.2 \text{ L}$

21. $n = 2 \Rightarrow h = \frac{2-0}{2} = 1 \Rightarrow \frac{h}{3} = \frac{1}{3}$;

$$\sum mf(x_i) = 12 \Rightarrow S = \frac{1}{3}(12) = 4;$$

$$\int_0^2 x^3 dx = \left[\frac{x^4}{4} \right]_0^2 = \frac{2^4}{4} - \frac{0^4}{4} = 4$$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	1	1	4	4
x_2	2	8	1	8

22. (a) $|E_S| \leq \frac{b-a}{180}(h^4)M$; $n = 4 \Rightarrow h = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$; $|f^{(4)}| \leq 1 \Rightarrow M = 1 \Rightarrow |E_S| \leq \frac{(\frac{\pi}{2}-0)}{180} \left(\frac{\pi}{8}\right)^4 (1) \approx 0.00021$

$$(b) h = \frac{\pi}{8} \Rightarrow \frac{h}{3} = \frac{\pi}{24};$$

$$\sum mf(x_i) = 10.472087048$$

$$\Rightarrow S = \frac{\pi}{24}(10.472087048) \approx 1.37079$$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	1	1	1
x_1	$\pi/8$	0.974495358	4	3.897981432
x_2	$\pi/4$	0.900316316	2	1.800632632
x_3	$3\pi/8$	0.784213303	4	3.136853212
x_4	$\pi/2$	0.636619772	1	0.636619772

$$(c) \approx \left(\frac{0.00021}{1.37079} \right) \times 100 \approx 0.015\%$$

$$23. (a) h = \frac{b-a}{n} = \frac{1-0}{10} = 0.1 \Rightarrow \operatorname{erf}(1) = \frac{2}{\sqrt{\pi}} \left(\frac{0.1}{3} \right) (y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 4y_9 + y_{10})$$

$$= \frac{2}{30\sqrt{\pi}} (e^0 + 4e^{-0.01} + 2e^{-0.04} + 4e^{-0.09} + \dots + 4e^{-0.81} + e^{-1}) \approx 0.8427$$

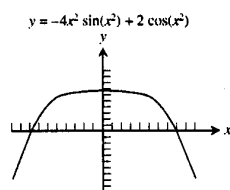
$$(b) |E_s| \leq \frac{1-0}{180} (0.1)^4 (12) \approx 6.7 \times 10^{-6}$$

24. The average of the 13 discrete temperatures gives equal weight to the low values at the end.

$$25. (a) f'(x) = 2x \cos(x^2)$$

$$f''(x) = 2x \cdot -2x \sin(x^2) + 2 \cos(x^2) = -4x^2 \sin(x^2) + 2 \cos(x^2)$$

(b)



(c) The graph shows that $-3 \leq f''(x) \leq 2$ so $|f''(x)| \leq 3$ for $-1 \leq x \leq 1$.

$$(d) |E_T| \leq \frac{1-(-1)}{12} (h^2)(3) = \frac{h^2}{2}$$

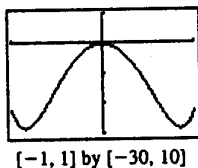
$$(e) \text{ For } 0 < h \leq 0.1, |E_T| \leq \frac{h^2}{2} \leq \frac{0.1^2}{2} = 0.005 < 0.01$$

$$(f) n \geq \frac{1-(-1)}{h} \geq \frac{2}{0.1} = 20$$

$$26. (a) f'''(x) = -4x^2 \cdot 2x \cos(x^2) - 8x \sin(x^2) - 4x \sin(x^2) = -8x^3 \cos(x^2) - 12x \sin(x^2)$$

$$f^{(4)}(x) = -8x^3 \cdot -2x \sin(x^2) - 24x^2 \cos(x^2) - 12x \cdot 2x \cos(x^2) - 12 \sin(x^2) \\ = (16x^4 - 12) \sin(x^2) - 48x^2 \cos(x^2)$$

(b)



(c) The graph shows that $-30 \leq f^{(4)}(x) \leq 0$ so $|f^{(4)}(x)| \leq 30$ for $-1 \leq x \leq 1$.

$$(d) |E_T| \leq \frac{1 - (-1)}{180} (h^4)(30) = \frac{h^4}{3}$$

$$(e) \text{ For } 0 < h \leq 0.4, |E_S| \leq \frac{h^4}{3} \leq \frac{0.4^4}{3} \approx 0.00853 < 0.01$$

$$(f) n \geq \frac{1 - (-1)}{h} \geq \frac{2}{0.4} = 5$$

Exercises 27-30 were done using the `nInt( )` command on a TI-92 Plus calculator. Your answer may be different depending on your calculator.

27. 3.1415927

28. 1.0894294

29. 1.3707622

30. 0.82811633

31. (a) $T_{10} \approx 1.983523538$

$$T_{100} \approx 1.999835504$$

$$T_{1000} \approx 1.999998355$$

n	$ E_T = 2 - T_n$
10	$0.016476462 = 1.6476462 \times 10^{-2}$
100	1.64496×10^{-4}
1000	1.645×10^{-6}

$$(c) |E_{T_{10n}}| \approx 10^{-2} |E_{T_n}|$$

$$(d) b - a = \pi, h^2 = \frac{\pi^2}{n^2}, M = 1 \Rightarrow |E_{T_n}| \leq \frac{\pi}{12} \left(\frac{\pi^2}{n^2} \right) = \frac{\pi^3}{12n^2} \text{ and } |E_{T_{10n}}| \leq \frac{\pi^3}{12(10n)^2} = 10^{-2} |E_{T_n}|$$

32. (a) $S_{10} \approx 2.000109517$

$$S_{100} \approx 2.000000011$$

$$S_{1000} \approx 2.000000000$$

(b)

n	$ E_S = 2 - S_n$
10	1.09517×10^{-4}
100	1.1×10^{-8}
1000	0

(c) $|E_{S_{10n}}| \approx 10^{-4} |E_{S_n}|$

(d) $b - a = \pi$, $h^4 = \frac{\pi^4}{n^4}$, $M = 1 \Rightarrow |E_{S_n}| \leq \frac{\pi}{180} \left(\frac{\pi^4}{n^4} \right) = \frac{\pi^5}{180n^4}$ and $|E_{S_{10n}}| \leq \frac{\pi^5}{180(10n)^4} = 10^{-4} |E_{S_n}|$

CHAPTER 4 PRACTICE EXERCISES

1. $\int (x^3 + 5x - 7) dx = \frac{x^4}{4} + \frac{5x^2}{2} - 7x + C$

2. $\int \left(8t^3 - \frac{t^2}{2} + t \right) dt = \frac{8t^4}{4} - \frac{t^3}{6} + \frac{t^2}{2} + C = 2t^4 - \frac{t^3}{6} + \frac{t^2}{2} + C$

3. $\int \left(3\sqrt{t} + \frac{4}{t^2} \right) dt = \int (3t^{1/2} + 4t^{-2}) dt = \frac{3t^{3/2}}{\left(\frac{3}{2}\right)} + \frac{4t^{-1}}{-1} + C = 2t^{3/2} - \frac{4}{t} + C$

4. $\int \left(\frac{1}{2\sqrt{t}} - \frac{3}{t^4} \right) dt = \int \left(\frac{1}{2}t^{-1/2} - 3t^{-4} \right) dt = \frac{1}{2} \left(\frac{t^{1/2}}{\frac{1}{2}} \right) - \frac{3t^{-3}}{(-3)} + C = \sqrt{t} + \frac{1}{t^3} + C$

5. Let $u = r^2 + 5 \Rightarrow du = 2r dr \Rightarrow \frac{1}{2} du = r dr$

$$\int \frac{r dr}{(r^2 + 5)^2} = \int \frac{\left(\frac{1}{2}\right) du}{u^2} = \frac{1}{2} \int u^{-2} du = \frac{1}{2} \left(\frac{u^{-1}}{-1} \right) + C = -\frac{1}{2} u^{-1} + C = -\frac{1}{2(r^2 + 5)} + C$$

6. Let $u = r^3 - \sqrt{2} \Rightarrow du = 3r^2 dr \Rightarrow 2 du = 6r^2 dr$

$$\int \frac{6r^2 dr}{(r^3 - \sqrt{2})^3} = \int \frac{2 du}{u^3} = 2 \int u^{-3} du = 2 \left(\frac{u^{-2}}{-2} \right) + C = -u^{-2} + C = -\frac{1}{(r^3 - \sqrt{2})^2} + C$$

7. Let $u = 2 - \theta^2 \Rightarrow du = -2\theta d\theta \Rightarrow -\frac{1}{2} du = \theta d\theta$

$$\int 3\theta\sqrt{2 - \theta^2} d\theta = \int \sqrt{u} \left(-\frac{3}{2} du \right) = -\frac{3}{2} \int u^{1/2} du = -\frac{3}{2} \left(\frac{u^{3/2}}{\frac{3}{2}} \right) + C = -u^{3/2} + C = -(2 - \theta^2)^{3/2} + C$$

8. Let $u = 73 + \theta^3 \Rightarrow du = 3\theta^2 d\theta \Rightarrow \frac{1}{27} du = \frac{\theta^2}{9} d\theta$

$$\int \frac{\theta^2}{9\sqrt{73+\theta^3}} d\theta = \int \frac{1}{\sqrt{u}} \left(\frac{1}{27} du \right) = \frac{1}{27} \int u^{-1/2} du = \frac{1}{27} \left(\frac{u^{1/2}}{\frac{1}{2}} \right) + C = \frac{2}{27} u^{1/2} + C = \frac{2}{27} \sqrt{73+\theta^3} + C$$

9. $\int e^x \sin(e^x) dx = \int \sin u du$, where $u = e^x$ and $du = e^x dx$
 $= -\cos u + C = -\cos(e^x) + C$

10. $\int e^t \cos(3e^t - 2) dt = \frac{1}{3} \int \cos u du$, where $u = 3e^t - 2$ and $du = 3e^t dt$
 $= \frac{1}{3} \sin u + C = \frac{1}{3} \sin(3e^t - 2) + C$

11. $\int e^x \sec^2(e^x - 7) dx = \int \sec^2 u du$, where $u = e^x - 7$ and $du = e^x dx$
 $= \tan u + C = \tan(e^x - 7) + C$

12. $\int e^y \csc(e^y + 1) \cot(e^y + 1) dy = \int \csc u \cot u du$, where $u = e^y + 1$ and $du = e^y dy$
 $= -\csc u + C = -\csc(e^y + 1) + C$

13. $\int \frac{\tan(\ln v)}{v} dv = \int \tan u du = \int \frac{\sin u}{\cos u} du$, where $u = \ln v$ and $du = \frac{1}{v} dv$
 $= -\ln |\cos u| + C = -\ln |\cos(\ln v)| + C$

14. Let $u = \frac{\theta}{3} \Rightarrow du = \frac{1}{3} d\theta \Rightarrow 3 du = d\theta$
 $\int \sec \frac{\theta}{3} \tan \frac{\theta}{3} d\theta = \int (\sec u \tan u)(3 du) = 3 \sec u + C = 3 \sec \frac{\theta}{3} + C$

15. Let $u = \frac{x}{4} \Rightarrow du = \frac{1}{4} dx \Rightarrow 4 du = dx$
 $\int \sin^2 \frac{x}{4} dx = \int (\sin^2 u)(4 du) = \int 4 \left(\frac{1 - \cos 2u}{2} \right) du = 2 \int (1 - \cos 2u) du = 2 \left(u - \frac{\sin 2u}{2} \right) + C$
 $= 2u - \sin 2u + C = 2 \left(\frac{x}{4} \right) - \sin 2 \left(\frac{x}{4} \right) + C = \frac{x}{2} - \sin \frac{x}{2} + C$

16. $\int \frac{(\ln x)^{-3}}{x} dx = \int u^{-3} du$, where $u = \ln x$ and $du = \frac{1}{x} dx$
 $= \frac{u^{-2}}{-2} + C = -\frac{1}{2} (\ln x)^{-2} + C$

17. $\int \frac{1}{r} \csc^2(1 + \ln r) dr = \int \csc^2 u du$, where $u = 1 + \ln r$ and $du = \frac{1}{r} dr$

$$= -\cot u + C = -\cot(1 + \ln r) + C$$

$$\begin{aligned} 18. \int \frac{\cos(1 - \ln v)}{v} dv &= - \int \cos u \, du, \text{ where } u = 1 - \ln v \text{ and } du = -\frac{1}{v} dv \\ &= -\sin u + C = -\sin(1 - \ln v) + C \end{aligned}$$

$$\begin{aligned} 19. \int x 3^{x^2} dx &= \frac{1}{2} \int 3^u du, \text{ where } u = x^2 \text{ and } du = 2x \, dx \\ &= \frac{1}{2 \ln 3} (3^u) + C = \frac{1}{2 \ln 3} (3^{x^2}) + C \end{aligned}$$

$$\begin{aligned} 20. \int 2^{\tan x} \sec^2 x \, dx &= \int 2^u du, \text{ where } u = \tan x \text{ and } du = \sec^2 x \, dx \\ &= \frac{1}{\ln 2} (2^u) + C = \frac{2^{\tan x}}{\ln 2} + C \end{aligned}$$

21. (a) Each time subinterval is of length $\Delta t = 0.4$ sec. The distance traveled over each subinterval, using the midpoint rule, is $\Delta h = \frac{1}{2}(v_i + v_{i+1})\Delta t$, where v_i is the velocity at the left, and v_{i+1} the velocity at the right, endpoint of the subinterval. We then add Δh to the height attained so far at the left endpoint v_i to arrive at the height associated with velocity v_{i+1} at the right endpoint. Using this methodology we build the following table based on the figure in the text:

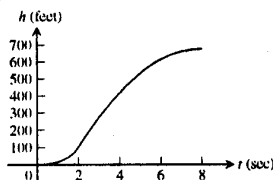
t (sec)	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0
v (fps)	0	10	25	55	100	190	180	170	155	140	130	120	105	90	80	65
h (ft)	0	2	9	25	56	114	188	258	323	382	436	486	531	570	604	633

t (sec)	6.4	6.8	7.2	7.6	8.0
v (fps)	52	40	30	15	0
h (ft)	656	674	688	697	700

NOTE: Your table values may vary slightly from ours depending on the v-values you read from the graph. Remember that some shifting of the graph occurs in the printing process.

The total height attained is about 700 ft.

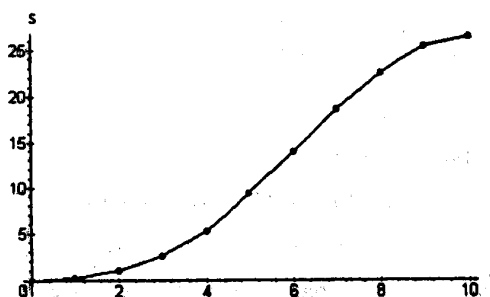
- (b) The graph is based on the table in part (a).



22. (a) Each time subinterval is of length $\Delta t = 1$ sec. The distance traveled over each subinterval, using the midpoint rule, is $\Delta s = \frac{1}{2}(v_i + v_{i+1})\Delta t$, where v_i is the velocity at the left, and v_{i+1} the velocity at the right, endpoint of the subinterval. We then add Δs to the distance attained so far at the left endpoint v_i to arrive at the distance associated with velocity v_{i+1} at the right endpoint. Using this methodology we build the table given below based on the figure in the text, obtaining approximately 26 m for the total distance traveled:

t (sec)	0	1	2	3	4	5	6	7	8	9	10
v (m/sec)	0	0.5	1.0	2	3.5	4.5	4.8	4.5	3.5	2	0
s (m)	0	0.25	1.00	2.5	5.25	9.25	13.9	18.55	22.55	25.3	26.3

- (b) The graph shows the distance traveled by the moving body as a function of time for $0 \leq t \leq 10$.



23. Let $u = 2x - 1 \Rightarrow du = 2 dx \Rightarrow \frac{1}{2} du = dx$; $x = 1 \Rightarrow u = 1$, $x = 5 \Rightarrow u = 9$

$$\int_1^5 (2x - 1)^{-1/2} dx = \int_1^9 u^{-1/2} \left(\frac{1}{2} du\right) = \left[u^{1/2}\right]_1^9 = 3 - 1 = 2$$

24. Let $u = x^2 - 1 \Rightarrow du = 2x dx \Rightarrow \frac{1}{2} du = x dx$; $x = 1 \Rightarrow u = 0$, $x = 3 \Rightarrow u = 8$

$$\int_1^3 x(x^2 - 1)^{1/3} dx = \int_0^8 u^{1/3} \left(\frac{1}{2} du\right) = \left[\frac{3}{8} u^{4/3}\right]_0^8 = \frac{3}{8}(16 - 0) = 6$$

25. Let $u = \frac{x}{2} \Rightarrow 2 du = dx$; $x = -\pi \Rightarrow u = -\frac{\pi}{2}$, $x = 0 \Rightarrow u = 0$

$$\int_{-\pi}^0 \cos\left(\frac{x}{2}\right) dx = \int_{-\pi/2}^0 (\cos u)(2 du) = [2 \sin u]_{-\pi/2}^0 = 2 \sin 0 - 2 \sin\left(-\frac{\pi}{2}\right) = 2(0 - (-1)) = 2$$

26. Let $u = \sin x \Rightarrow du = \cos x dx$; $x = 0 \Rightarrow u = 0$, $x = \frac{\pi}{2} \Rightarrow u = 1$

$$\int_0^{\pi/2} (\sin x)(\cos x) dx = \int_0^1 u du = \left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2}$$

$$27. (a) \int_{-2}^2 f(x) dx = \frac{1}{3} \int_{-2}^2 3 f(x) dx = \frac{1}{3}(12) = 4 \quad (b) \int_2^5 f(x) dx = \int_{-2}^5 f(x) dx - \int_{-2}^2 f(x) dx = 6 - 4 = 2$$

$$(c) \int_5^{-2} g(x) dx = - \int_{-2}^5 g(x) dx = -2 \quad (d) \int_{-2}^5 (-\pi g(x)) dx = -\pi \int_{-2}^5 g(x) dx = -\pi(2) = -2\pi$$

$$(e) \int_{-2}^5 \left(\frac{f(x) + g(x)}{5} \right) dx = \frac{1}{5} \int_{-2}^5 f(x) dx + \frac{1}{5} \int_{-2}^5 g(x) dx = \frac{1}{5}(6) + \frac{1}{5}(2) = \frac{8}{5}$$

$$28. (a) \int_0^2 g(x) dx = \frac{1}{7} \int_0^2 7 g(x) dx = \frac{1}{7}(7) = 1 \quad (b) \int_1^2 g(x) dx = \int_0^2 g(x) dx - \int_0^1 g(x) dx = 1 - 2 = -1$$

$$(c) \int_2^0 f(x) dx = - \int_0^2 f(x) dx = -\pi \quad (d) \int_0^2 \sqrt{2} f(x) dx = \sqrt{2} \int_0^2 f(x) dx = \sqrt{2}(\pi) = \pi\sqrt{2}$$

$$(e) \int_0^2 [g(x) - 3 f(x)] dx = \int_0^2 g(x) dx - 3 \int_0^2 f(x) dx = 1 - 3\pi$$

$$29. \int_{-1}^1 (3x^2 - 4x + 7) dx = [x^3 - 2x^2 + 7x]_{-1}^1 = [1^3 - 2(1)^2 + 7(1)] - [(-1)^3 - 2(-1)^2 + 7(-1)] = 6 - (-10) = 16$$

$$30. \int_0^1 (8s^3 - 12s^2 + 5) ds = [2s^4 - 4s^3 + 5s]_0^1 = [2(1)^4 - 4(1)^3 + 5(1)] - 0 = 3$$

$$31. \int_1^4 \left(\frac{x}{8} + \frac{1}{2x} \right) dx = \frac{1}{2} \int_1^4 \left(\frac{1}{4}x + \frac{1}{x} \right) dx = \frac{1}{2} \left[\frac{1}{8}x^2 + \ln |x| \right]_1^4 = \frac{1}{2} \left[\left(\frac{16}{8} + \ln 4 \right) - \left(\frac{1}{8} + \ln 1 \right) \right] = \frac{15}{16} + \frac{1}{2} \ln 4$$

$$= \frac{15}{16} + \ln \sqrt{4} = \frac{15}{16} + \ln 2$$

$$32. \int_1^8 \left(\frac{2}{3x} - \frac{8}{x^2} \right) dx = \frac{2}{3} \int_1^8 \left(\frac{1}{x} - 12x^{-2} \right) dx = \frac{2}{3} [\ln |x| + 12x^{-1}]_1^8 = \frac{2}{3} \left[\left(\ln 8 + \frac{12}{8} \right) - (\ln 1 + 12) \right]$$

$$= \frac{2}{3} \left(\ln 8 + \frac{3}{2} - 12 \right) = \frac{2}{3} \left(\ln 8 - \frac{21}{2} \right) = \frac{2}{3} (\ln 8) - 7 = \ln(8^{2/3}) - 7 = \ln 4 - 7$$

$$33. \int_1^4 \frac{dt}{t\sqrt{t}} = \int_1^4 \frac{dt}{t^{3/2}} = \int_1^4 t^{-3/2} dt = [-2t^{-1/2}]_1^4 = \frac{-2}{\sqrt{4}} - \frac{(-2)}{\sqrt{1}} = 1$$

$$34. \text{ Let } x = 1 + \sqrt{u} \Rightarrow dx = \frac{1}{2}u^{-1/2} du \Rightarrow 2 dx = \frac{du}{\sqrt{u}}; u = 1 \Rightarrow x = 2, u = 4 \Rightarrow x = 3$$

$$\int_1^4 \frac{(1+\sqrt{u})^{1/2}}{\sqrt{u}} du = \int_2^3 x^{1/2}(2 dx) = \left[2\left(\frac{2}{3}\right)x^{3/2}\right]_2^3 = \frac{4}{3}(3^{3/2}) - \frac{4}{3}(2^{3/2}) = 4\sqrt{3} - \frac{8}{3}\sqrt{2} = \frac{4}{3}(3\sqrt{3} - 2\sqrt{2})$$

35. Let $u = 2x + 1 \Rightarrow du = 2 dx \Rightarrow 18 du = 36 dx$; $x = 0 \Rightarrow u = 1$, $x = 1 \Rightarrow u = 3$

$$\int_0^1 \frac{36 dx}{(2x+1)^3} = \int_1^3 18u^{-3} du = \left[\frac{18u^{-2}}{-2}\right]_1^3 = \left[\frac{-9}{u^2}\right]_1^3 = \left(\frac{-9}{3^2}\right) - \left(\frac{-9}{1^2}\right) = 8$$

36. Let $u = 7 - 5r \Rightarrow du = -5 dr \Rightarrow -\frac{1}{5} du = dr$; $r = 0 \Rightarrow u = 7$, $r = 1 \Rightarrow u = 2$

$$\int_0^1 \frac{dr}{\sqrt[3]{(7-5r)^2}} = \int_7^2 (7-5r)^{-2/3} dr = \int_7^2 u^{-2/3} \left(-\frac{1}{5} du\right) = -\frac{1}{5} [3u^{1/3}]_7^2 = \frac{3}{5} (3\sqrt[3]{7} - 3\sqrt[3]{2})$$

37. $\int_0^{\ln 5} e^r (3e^r + 1)^{-3/2} dr = \frac{1}{3} \int_4^{16} u^{-3/2} du$, where $u = 3e^r + 1$, $du = 3e^r dr$; $r = 0 \Rightarrow u = 4$, $r = \ln 5 \Rightarrow u = 16$

$$= -\frac{2}{3} [u^{-1/2}]_4^{16} = -\frac{2}{3} (16^{-1/2} - 4^{-1/2}) = \left(-\frac{2}{3}\right) \left(\frac{1}{4} - \frac{1}{2}\right) = \left(-\frac{2}{3}\right) \left(-\frac{1}{4}\right) = \frac{1}{6}$$

38. $\int_0^{\ln 9} e^\theta (e^\theta - 1)^{1/2} d\theta = \int_0^8 u^{1/2} du$, where $u = e^\theta - 1$, $du = e^\theta d\theta$; $\theta = 0 \Rightarrow u = 0$, $\theta = \ln 9 \Rightarrow u = 8$

$$= \frac{2}{3} [u^{3/2}]_0^8 = \frac{2}{3} (8^{3/2} - 0^{3/2}) = \frac{2}{3} (2^{9/2} - 0) = \frac{2^{11/2}}{3} = \frac{32\sqrt{2}}{3}$$

39. $\int_e^{e^2} \frac{1}{x\sqrt{\ln x}} dx = \int_e^{e^2} (\ln x)^{-1/2} \frac{1}{x} dx = \int_1^2 u^{-1/2} du$, where $u = \ln x$, $du = \frac{1}{x} dx$; $x = e \Rightarrow u = 1$, $x = e^2 \Rightarrow u = 2$

$$= 2[u^{1/2}]_1^2 = 2(\sqrt{2} - 1) = 2\sqrt{2} - 2$$

40. Let $u = 4t - \frac{\pi}{4} \Rightarrow du = 4 dt \Rightarrow \frac{1}{4} du = dt$; $t = 0 \Rightarrow u = -\frac{\pi}{4}$, $t = \frac{\pi}{4} \Rightarrow u = \frac{3\pi}{4}$

$$\int_0^{\pi/4} \cos^2\left(4t - \frac{\pi}{4}\right) dt = \int_{-\pi/4}^{3\pi/4} (\cos^2 u) \left(\frac{1}{4} du\right) = \frac{1}{4} \left[\frac{u}{2} + \frac{\sin 2u}{4}\right]_{-\pi/4}^{3\pi/4} = \frac{1}{4} \left(\frac{3\pi}{8} + \frac{\sin\left(\frac{3\pi}{2}\right)}{4}\right) - \frac{1}{4} \left(-\frac{\pi}{8} + \frac{\sin\left(-\frac{\pi}{2}\right)}{4}\right)$$

$$= \frac{\pi}{8} - \frac{1}{16} + \frac{1}{16} = \frac{\pi}{8}$$

41. $\int_1^e \frac{1}{x} (1 + 7 \ln x)^{-1/3} dx = \frac{1}{7} \int_1^8 u^{-1/3} du$, where $u = 1 + 7 \ln x$, $du = \frac{7}{x} dx$; $x = 1 \Rightarrow u = 1$, $x = e \Rightarrow u = 8$

$$= \frac{3}{14} [u^{2/3}]_1^8 = \frac{3}{14} (8^{2/3} - 1^{2/3}) = \left(\frac{3}{14}\right) (4 - 1) = \frac{9}{14}$$

$$42. \int_{\pi/4}^{3\pi/4} \csc^2 x \, dx = [-\cot x]_{\pi/4}^{3\pi/4} = \left(-\cot \frac{3\pi}{4}\right) - \left(-\cot \frac{\pi}{4}\right) = 2$$

$$43. \int_{-2}^2 \frac{3}{4+3t^2} \, dt = \sqrt{3} \int_{-2}^2 \frac{\sqrt{3}}{2^2 + (\sqrt{3}t)^2} \, dt = \sqrt{3} \int_{-2\sqrt{3}}^{2\sqrt{3}} \frac{1}{2^2 + u^2} \, du, \text{ where } u = \sqrt{3}t, \, du = \sqrt{3} \, dt;$$

$$t = -2 \Rightarrow u = -2\sqrt{3}, \, t = 2 \Rightarrow u = 2\sqrt{3}$$

$$= \sqrt{3} \left[\frac{1}{2} \tan^{-1} \left(\frac{u}{2} \right) \right]_{-2\sqrt{3}}^{2\sqrt{3}} = \frac{\sqrt{3}}{2} [\tan^{-1}(\sqrt{3}) - \tan^{-1}(-\sqrt{3})] = \frac{\sqrt{3}}{2} \left[\frac{\pi}{3} - \left(-\frac{\pi}{3} \right) \right] = \frac{\pi}{\sqrt{3}}$$

$$44. \int_1^3 \frac{[\ln(v+1)]^2}{v+1} \, dv = \int_1^3 [\ln(v+1)]^2 \frac{1}{v+1} \, dv = \int_{\ln 2}^{\ln 4} u^2 \, du, \text{ where } u = \ln(v+1), \, du = \frac{1}{v+1} \, dv;$$

$$v = 1 \Rightarrow u = \ln 2, \, v = 3 \Rightarrow u = \ln 4;$$

$$= \frac{1}{3} [u^3]_{\ln 2}^{\ln 4} = \frac{1}{3} [(\ln 4)^3 - (\ln 2)^3] = \frac{1}{3} [(2 \ln 2)^3 - (\ln 2)^3] = \frac{(\ln 2)^3}{3} (8 - 1) = \frac{7}{3} (\ln 2)^3$$

$$45. \int_{-\pi/3}^0 \sec x \tan x \, dx = [\sec x]_{-\pi/3}^0 = \sec 0 - \sec \left(-\frac{\pi}{3} \right) = 1 - 2 = -1$$

$$46. \int_{\pi/4}^{3\pi/4} \csc z \cot z \, dz = [-\csc z]_{\pi/4}^{3\pi/4} = \left(-\csc \frac{3\pi}{4} \right) - \left(-\csc \frac{\pi}{4} \right) = -\sqrt{2} + \sqrt{2} = 0$$

$$47. \int_{\sqrt{2}/3}^{2/3} \frac{1}{|y| \sqrt{9y^2 - 1}} \, dy = \int_{\sqrt{2}/3}^{2/3} \frac{3}{|3y| \sqrt{(3y)^2 - 1}} \, dy = \int_{\sqrt{2}}^2 \frac{1}{|u| \sqrt{u^2 - 1}} \, du, \text{ where } u = 3y, \, du = 3 \, dy;$$

$$y = \frac{\sqrt{2}}{3} \Rightarrow u = \sqrt{2}, \, y = \frac{2}{3} \Rightarrow u = 2$$

$$= [\sec^{-1} u]_{\sqrt{2}}^2 = [\sec^{-1} 2 - \sec^{-1} \sqrt{2}] = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

$$48. \text{ Let } u = 1 - x^2 \Rightarrow du = -2x \, dx \Rightarrow -du = 2x \, dx; \, x = -1 \Rightarrow u = 0, \, x = 1 \Rightarrow u = 0$$

$$\int_{-1}^1 2x \sin(1 - x^2) \, dx = \int_0^0 -\sin u \, du = 0$$

$$49. \text{ Let } u = 1 + 3 \sin^2 x \Rightarrow du = 6 \sin x \cos x \, dx \Rightarrow \frac{1}{2} du = 3 \sin x \cos x \, dx; \, x = 0 \Rightarrow u = 1, \, x = \frac{\pi}{2} \Rightarrow u = 1 + 3 \sin^2 \frac{\pi}{2} = 4$$

$$\int_0^{\pi/2} \frac{3 \sin x \cos x}{\sqrt{1 + 3 \sin^2 x}} \, dx = \int_1^4 \frac{1}{\sqrt{u}} \left(\frac{1}{2} du \right) = \int_1^4 \frac{1}{2} u^{-1/2} \, du = \left[\frac{1}{2} \left(\frac{u^{1/2}}{\frac{1}{2}} \right) \right]_1^4 = [u^{1/2}]_1^4 = 4^{1/2} - 1^{1/2} = 1$$

$$50. \int_{-2}^{-1} \frac{2}{v^2 + 4v + 5} dv = 2 \int_{-2}^{-1} \frac{1}{1 + (v^2 + 4v + 4)} dv = 2 \int_{-2}^{-1} \frac{1}{1 + (v+2)^2} dv = 2 \int_0^1 \frac{1}{1+u^2} du,$$

where $u = v + 2$, $du = dv$; $v = -2 \Rightarrow u = 0$, $v = -1 \Rightarrow u = 1$

$$= 2[\tan^{-1} u]_0^1 = 2(\tan^{-1} 1 - \tan^{-1} 0) = 2\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{2}$$

$$51. \int_{-1}^1 \frac{3}{4v^2 + 4v + 4} dv = \frac{3}{4} \int_{-1}^1 \frac{1}{\frac{3}{4} + (v^2 + v + \frac{1}{4})} dv = \frac{3}{4} \int_{-1}^1 \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(v + \frac{1}{2}\right)^2} dv = \frac{3}{4} \int_{-1/2}^{3/2} \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2 + u^2} du$$

where $u = v + \frac{1}{2}$, $du = dv$; $v = -1 \Rightarrow u = -\frac{1}{2}$, $v = 1 \Rightarrow u = \frac{3}{2}$

$$= \frac{3}{4} \left[\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2u}{\sqrt{3}} \right) \right]_{-1/2}^{3/2} = \frac{\sqrt{3}}{2} \left[\tan^{-1} \sqrt{3} - \tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) \right] = \frac{\sqrt{3}}{2} \left[\frac{\pi}{3} - \left(-\frac{\pi}{6} \right) \right] = \frac{\sqrt{3}}{2} \left(\frac{2\pi}{6} + \frac{\pi}{6} \right) = \frac{\sqrt{3}}{2} \cdot \frac{\pi}{2}$$

$$= \frac{\sqrt{3}\pi}{4}$$

$$52. \text{ Let } u = \sin \sqrt{t} \Rightarrow du = (\cos \sqrt{t}) \left(\frac{1}{2} t^{-1/2} \right) dt = \frac{\cos \sqrt{t}}{2\sqrt{t}} dt \Rightarrow 2 du = \frac{\cos \sqrt{t}}{\sqrt{t}} dt; t = \frac{\pi^2}{36} \Rightarrow u = \sin \frac{\pi}{6} = \frac{1}{2},$$

$$t = \frac{\pi^2}{4} \Rightarrow u = \sin \frac{\pi}{2} = 1$$

$$\int_{\pi^2/36}^{\pi^2/4} \frac{\cos \sqrt{t}}{\sqrt{t} \sin \sqrt{t}} dt = \int_{1/2}^1 \frac{1}{\sqrt{u}} (2 du) = 2 \int_{1/2}^1 u^{-1/2} du = [4\sqrt{u}]_{1/2}^1 = 4\sqrt{1} - 4\sqrt{1/2} = 2(2 - \sqrt{2})$$

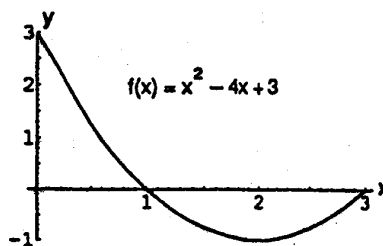
$$53. x^2 - 4x + 3 = 0 \Rightarrow (x-3)(x-1) = 0 \Rightarrow x = 3 \text{ or } x = 1;$$

$$\text{Area} = \int_0^1 (x^2 - 4x + 3) dx - \int_1^3 (x^2 - 4x + 3) dx$$

$$= \left[\frac{x^3}{3} - 2x^2 + 3x \right]_0^1 - \left[\frac{x^3}{3} - 2x^2 + 3x \right]_1^3$$

$$= \left[\left(\frac{1^3}{3} - 2(1)^2 + 3(1) \right) - 0 \right]$$

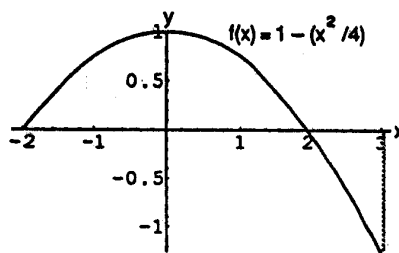
$$- \left[\left(\frac{3^3}{3} - 2(3)^2 + 3(3) \right) - \left(\frac{1^3}{3} - 2(1)^2 + 3(1) \right) \right] = \left(\frac{1}{3} + 1 \right) - \left[0 - \left(\frac{1}{3} + 1 \right) \right] = \frac{8}{3}$$



$$54. 1 - \frac{x^2}{4} = 0 \Rightarrow 4 - x^2 = 0 \Rightarrow x = \pm 2;$$

$$\text{Area} = \int_{-2}^2 \left(1 - \frac{x^2}{4} \right) dx - \int_2^3 \left(1 - \frac{x^2}{4} \right) dx$$

$$= \left[x - \frac{x^3}{12} \right]_{-2}^2 - \left[x - \frac{x^3}{12} \right]_2^3$$



$$\begin{aligned}
 &= \left[\left(2 - \frac{2^3}{12} \right) - \left(-2 - \frac{(-2)^3}{12} \right) \right] \\
 &\quad - \left[\left(3 - \frac{3^3}{12} \right) - \left(2 - \frac{2^3}{12} \right) \right] = \left[\frac{4}{3} - \left(-\frac{4}{3} \right) \right] - \left(\frac{3}{4} - \frac{4}{3} \right) = \frac{13}{4}
 \end{aligned}$$

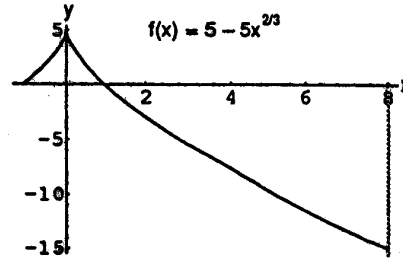
$$55. 5 - 5x^{2/3} = 0 \Rightarrow 1 - x^{2/3} = 0 \Rightarrow x = \pm 1;$$

$$\text{Area} = \int_{-1}^1 (5 - 5x^{2/3}) dx - \int_1^8 (5 - 5x^{2/3}) dx$$

$$= [5x - 3x^{5/3}]_{-1}^1 - [5x - 3x^{5/3}]_1^8$$

$$= [(5(1) - 3(1)^{5/3}) - (5(-1) - 3(-1)^{5/3})]$$

$$- [(5(8) - 3(8)^{5/3}) - (5(1) - 3(1)^{5/3})] = [2 - (-2)] - [(40 - 96) - 2] = 62$$



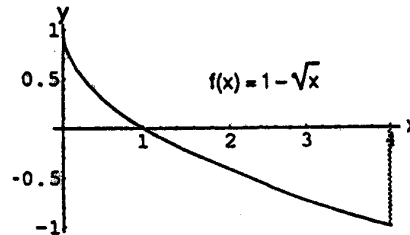
$$56. 1 - \sqrt{x} = 0 \Rightarrow x = 1;$$

$$\text{Area} = \int_0^1 (1 - \sqrt{x}) dx - \int_1^4 (1 - \sqrt{x}) dx$$

$$= \left[x - \frac{2}{3}x^{3/2} \right]_0^1 - \left[x - \frac{2}{3}x^{3/2} \right]_1^4$$

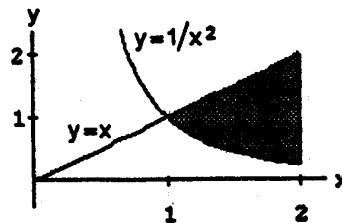
$$= \left[\left(1 - \frac{2}{3}(1)^{3/2} \right) - 0 \right] - \left[\left(4 - \frac{2}{3}(4)^{3/2} \right) - \left(1 - \frac{2}{3}(1)^{3/2} \right) \right]$$

$$= \frac{1}{3} - \left[\left(4 - \frac{16}{3} \right) - \frac{1}{3} \right] = 2$$



$$57. f(x) = x, g(x) = \frac{1}{x^2}, a = 1, b = 2 \Rightarrow A = \int_a^b [f(x) - g(x)] dx$$

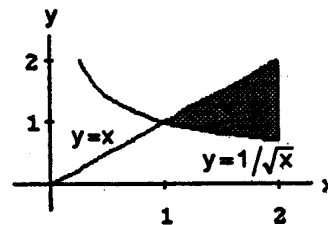
$$= \int_1^2 \left(x - \frac{1}{x^2} \right) dx = \left[\frac{x^2}{2} + \frac{1}{x} \right]_1^2 = \left(\frac{4}{2} + \frac{1}{2} \right) - \left(\frac{1}{2} + 1 \right) = 1$$



$$58. f(x) = x, g(x) = \frac{1}{\sqrt{x}}, a = 1, b = 2 \Rightarrow A = \int_a^b [f(x) - g(x)] dx$$

$$= \int_1^2 \left(x - \frac{1}{\sqrt{x}} \right) dx = \left[\frac{x^2}{2} - 2\sqrt{x} \right]_1^2 = \left(\frac{4}{2} - 2\sqrt{2} \right) - \left(\frac{1}{2} - 2 \right)$$

$$= \frac{7 - 4\sqrt{2}}{2}$$

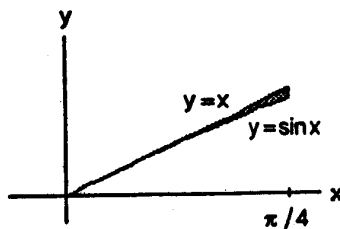


$$\begin{aligned}
 59. \quad f(x) &= (1 - \sqrt{x})^2, \quad g(x) = 0, \quad a = 0, \quad b = 1 \Rightarrow A = \int_a^b [f(x) - g(x)] \, dx = \int_0^1 (1 - \sqrt{x})^2 \, dx = \int_0^1 (1 - 2\sqrt{x} + x) \, dx \\
 &= \int_0^1 (1 - 2x^{1/2} + x) \, dx = \left[x - \frac{4}{3}x^{3/2} + \frac{x^2}{2} \right]_0^1 = 1 - \frac{4}{3} + \frac{1}{2} = \frac{1}{6}(6 - 8 + 3) = \frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 60. \quad f(x) &= (1 - x^3)^2, \quad g(x) = 0, \quad a = 0, \quad b = 1 \Rightarrow A = \int_a^b [f(x) - g(x)] \, dx = \int_0^1 (1 - x^3)^2 \, dx = \int_0^1 (1 - 2x^3 + x^6) \, dx \\
 &= \left[x - \frac{x^4}{2} + \frac{x^7}{7} \right]_0^1 = 1 - \frac{1}{2} + \frac{1}{7} = \frac{9}{14}
 \end{aligned}$$

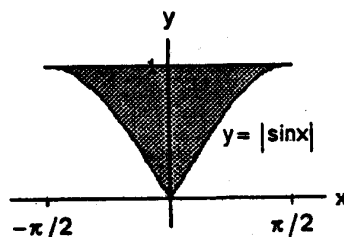
$$61. \quad f(x) = x, \quad g(x) = \sin x, \quad a = 0, \quad b = \frac{\pi}{4}$$

$$\begin{aligned}
 \Rightarrow A &= \int_a^b [f(x) - g(x)] \, dx = \int_0^{\pi/4} (x - \sin x) \, dx \\
 &= \left[\frac{x^2}{2} + \cos x \right]_0^{\pi/4} = \left(\frac{\pi^2}{32} + \frac{\sqrt{2}}{2} \right) - 1
 \end{aligned}$$



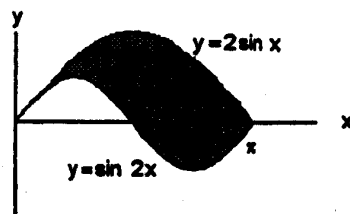
$$62. \quad f(x) = 1, \quad g(x) = |\sin x|, \quad a = -\frac{\pi}{2}, \quad b = \frac{\pi}{2}$$

$$\begin{aligned}
 \Rightarrow A &= \int_a^b [f(x) - g(x)] \, dx = \int_{-\pi/2}^{\pi/2} (1 - |\sin x|) \, dx \\
 &= \int_{-\pi/2}^0 (1 + \sin x) \, dx + \int_0^{\pi/2} (1 - \sin x) \, dx \\
 &= 2 \int_0^{\pi/2} (1 - \sin x) \, dx = 2[x + \cos x]_0^{\pi/2} = 2\left(\frac{\pi}{2} - 1\right) = \pi - 2
 \end{aligned}$$



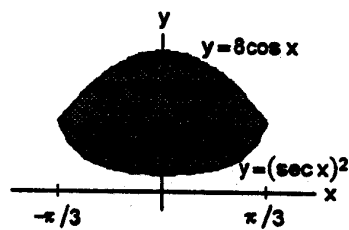
$$63. \quad a = 0, \quad b = \pi, \quad f(x) - g(x) = 2 \sin x - \sin 2x$$

$$\begin{aligned}
 \Rightarrow A &= \int_0^{\pi} (2 \sin x - \sin 2x) \, dx = \left[-2 \cos x + \frac{\cos 2x}{2} \right]_0^{\pi} \\
 &= \left[-2 \cdot (-1) + \frac{1}{2} \right] - \left(-2 \cdot 1 + \frac{1}{2} \right) = 4
 \end{aligned}$$



64. $a = -\frac{\pi}{3}$, $b = \frac{\pi}{3}$, $f(x) - g(x) = 8 \cos x - \sec^2 x$

$$\begin{aligned} \Rightarrow A &= \int_{-\pi/3}^{\pi/3} (8 \cos x - \sec^2 x) dx = [8 \sin x - \tan x]_{-\pi/3}^{\pi/3} \\ &= \left(8 \cdot \frac{\sqrt{3}}{2} - \sqrt{3}\right) - \left(-8 \cdot \frac{\sqrt{3}}{2} + \sqrt{3}\right) = 6\sqrt{3} \end{aligned}$$



65. $f(x) = x^3 - 3x^2 = x^2(x - 3) \Rightarrow f'(x) = 3x^2 - 6x = 3x(x - 2) \Rightarrow f' = +++ \mid \text{----} \mid +++$

$$\begin{aligned} \Rightarrow f(0) = 0 \text{ is a maximum and } f(2) = -4 \text{ is a minimum. Then } A &= - \int_0^3 (x^3 - 3x^2) dx = - \left[\frac{x^4}{4} - x^3 \right]_0^3 \\ &= - \left(\frac{81}{4} - 27 \right) = \frac{27}{4} \end{aligned}$$

66. $A = \int_0^1 (1 - x^{1/3})^3 dx = \int_0^1 (1 - 3x^{1/3} + 3x^{2/3} - x) dx = \left[x - \frac{9}{4}x^{4/3} + \frac{9}{5}x^{5/3} - \frac{x^2}{2} \right]_0^1 = \frac{1}{20}$

67. $y = \int \frac{x^2 + 1}{x^2} dx = \int (1 + x^{-2}) dx = x - x^{-1} + C = x - \frac{1}{x} + C$; $y = -1$ when $x = 1 \Rightarrow 1 - \frac{1}{1} + C = -1$
 $\Rightarrow C = -1 \Rightarrow y = x - \frac{1}{x} - 1$

68. $\frac{dy}{dx} = e^{-x-y-2} \Rightarrow \frac{dy}{dx} = e^{-x-2} \cdot e^{-y} \Rightarrow e^y dy = e^{-x-2} dx = \int e^y dy = \int e^{-x-2} dx \Rightarrow e^y = -e^{-x-2} + C$; $x = 0$
 and $y = -2 \Rightarrow e^{-2} = -e^{-2} + C \Rightarrow C = 2e^{-2} \Rightarrow e^y = -e^{-x-2} + 2e^{-2} \Rightarrow \ln(e^y) = \ln(-e^{-x-2} + 2e^{-2})$
 $\Rightarrow y = \ln(-e^{-x-2} + 2e^{-2})$

69. $\frac{dr}{dt} = \int \left(15\sqrt{t} + \frac{3}{\sqrt{t}} \right) dt = \int (15t^{1/2} + 3t^{-1/2}) dt = 10t^{3/2} + 6t^{1/2} + C$; $\frac{dr}{dt} = 8$ when $t = 1$
 $\Rightarrow 10(1)^{3/2} + 6(1)^{1/2} + C = 8 \Rightarrow C = -8$. Thus $\frac{dr}{dt} = 10t^{3/2} + 6t^{1/2} - 8 \Rightarrow r = \int (10t^{3/2} + 6t^{1/2} - 8) dt$
 $= 4t^{5/2} + 4t^{3/2} - 8t + C$; $r = 0$ when $t = 1 \Rightarrow 4(1)^{5/2} + 4(1)^{3/2} - 8(1) + C_1 = 0 \Rightarrow C_1 = 0$. Therefore,
 $r = 4t^{5/2} + 4t^{3/2} - 8t$

70. $\frac{d^2r}{dt^2} = \int -\cos t dt = -\sin t + C$; $r'' = 0$ when $t = 0 \Rightarrow -\sin 0 + C = 0 \Rightarrow C = 0$. Thus, $\frac{d^2r}{dt^2} = -\sin t$
 $\Rightarrow \frac{dr}{dt} = \int -\sin t dt = \cos t + C_1$; $r' = 0$ when $t = 0 \Rightarrow 1 + C_1 = 0 \Rightarrow C_1 = -1$. Then $\frac{dr}{dt} = \cos t - 1$
 $\Rightarrow r = \int (\cos t - 1) dt = \sin t - t + C_2$; $r = -1$ when $t = 0 \Rightarrow 0 - 0 + C_2 = -1$. Therefore, $r = \sin t - t - 1$

$$71. y = x^2 + \int_1^x \frac{1}{t} dt \Rightarrow \frac{dy}{dx} = 2x + \frac{1}{x} \Rightarrow \frac{d^2y}{dx^2} = 2 - \frac{1}{x^2}; y(1) = 1 + \int_1^1 \frac{1}{t} dt = 1 \text{ and } y'(1) = 2 + 1 = 3$$

$$72. y = \int_0^x (1 + 2\sqrt{\sec t}) dt \Rightarrow \frac{dy}{dx} = 1 + 2\sqrt{\sec x} \Rightarrow \frac{d^2y}{dx^2} = 2\left(\frac{1}{2}\right)(\sec x)^{-1/2}(\sec x \tan x) = \sqrt{\sec x}(\tan x);$$

$$x = 0 \Rightarrow y = \int_0^0 (1 + 2\sqrt{\sec t}) dt = 0 \text{ and } x = 0 \Rightarrow \frac{dy}{dx} = 1 + 2\sqrt{\sec 0} = 3$$

$$73. y = \int_5^x \frac{\sin t}{t} dt - 3 \Rightarrow \frac{dy}{dx} = \frac{\sin x}{x}; x = 5 \Rightarrow y = \int_5^5 \frac{\sin t}{t} dt - 3 = -3$$

$$74. y = \int_{-1}^x \sqrt{2 - \sin^2 t} dt + 2 \text{ so that } \frac{dy}{dx} = \sqrt{2 - \sin^2 x}; x = -1 \Rightarrow y = \int_{-1}^{-1} \sqrt{2 - \sin^2 t} dt + 2 = 2$$

$$75. (a) \text{ av}(f) = \frac{1}{1 - (-1)} \int_{-1}^1 (mx + b) dx = \frac{1}{2} \left[\frac{mx^2}{2} + bx \right]_{-1}^1 = \frac{1}{2} \left[\left(\frac{m(1)^2}{2} + b(1) \right) - \left(\frac{m(-1)^2}{2} + b(-1) \right) \right] = \frac{1}{2}(2b) = b$$

$$(b) \text{ av}(f) = \frac{1}{k - (-k)} \int_{-k}^k (mx + b) dx = \frac{1}{2k} \left[\frac{mx^2}{2} + bx \right]_{-k}^k = \frac{1}{2k} \left[\left(\frac{m(k)^2}{2} + b(k) \right) - \left(\frac{m(-k)^2}{2} + b(-k) \right) \right]$$

$$= \frac{1}{2k}(2bk) = b$$

$$76. (a) y_{\text{av}} = \frac{1}{3 - 0} \int_0^3 \sqrt{3x} dx = \frac{1}{3} \int_0^3 \sqrt{3} x^{1/2} dx = \frac{\sqrt{3}}{3} \left[\frac{2}{3} x^{3/2} \right]_0^3 = \frac{\sqrt{3}}{3} \left[\frac{2}{3} (3)^{3/2} - \frac{2}{3} (0)^{3/2} \right] = \frac{\sqrt{3}}{3} (2\sqrt{3}) = 2$$

$$(b) y_{\text{av}} = \frac{1}{a - 0} \int_0^a \sqrt{ax} dx = \frac{1}{a} \int_0^a \sqrt{a} x^{1/2} dx = \frac{\sqrt{a}}{a} \left[\frac{2}{3} x^{3/2} \right]_0^a = \frac{\sqrt{a}}{a} \left(\frac{2}{3} (a)^{3/2} - \frac{2}{3} (0)^{3/2} \right) = \frac{\sqrt{a}}{a} \left(\frac{2}{3} a \sqrt{a} \right) = \frac{2}{3} a$$

$$77. f'_{\text{av}} = \frac{1}{b - a} \int_a^b f'(x) dx = \frac{1}{b - a} [f(x)]_a^b = \frac{1}{b - a} [f(b) - f(a)] = \frac{f(b) - f(a)}{b - a} \text{ so the average value of } f' \text{ over } [a, b] \text{ is the}$$

slope of the secant line joining the points $(a, f(a))$ and $(b, f(b))$.

$$78. \text{ Yes, because the average value of } f \text{ on } [a, b] \text{ is } \frac{1}{b - a} \int_a^b f(x) dx. \text{ If the length of the interval is 2, then } b - a = 2$$

and the average value of the function is $\frac{1}{2} \int_a^b f(x) dx$.

$$79. \frac{dy}{dx} = \sqrt{2 + \cos^3 x}$$

$$80. \frac{dy}{dx} = \sqrt{2 + \cos^3(7x^2)} \cdot \frac{d}{dx}(7x^2) = 14x \sqrt{2 + \cos^3(7x^2)}$$

$$81. \frac{dy}{dx} = \frac{d}{dx} \left(- \int_1^x \frac{6}{3+t^4} dt \right) = - \frac{6}{3+x^4}$$

$$82. \frac{dy}{dx} = \frac{d}{dx} \left(\int_0^{e^{2x}} \frac{1}{1+t^2} dt - \int_0^x \frac{1}{1+t^2} dt \right) = \frac{1}{1+(e^{2x})^2} (2e^{2x}) - \frac{1}{1+x^2} = \frac{2e^{2x}}{1+e^{4x}} - \frac{1}{1+x^2}$$

$$83. h = \frac{b-a}{n} = \frac{\pi-0}{6} = \frac{\pi}{6} \Rightarrow \frac{h}{2} = \frac{\pi}{12};$$

$$\sum_{i=0}^6 mf(x_i) = 12 \Rightarrow T = \left(\frac{\pi}{12} \right) (12) = \pi;$$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	$\pi/6$	$1/2$	2	1
x_2	$\pi/3$	$3/2$	2	3
x_3	$\pi/2$	2	2	4
x_4	$2\pi/3$	$3/2$	2	3
x_5	$5\pi/6$	$1/2$	2	1
x_6	π	0	1	0

$$\sum_{i=0}^6 mf(x_i) = 18 \text{ and } \frac{h}{3} = \frac{\pi}{18} \Rightarrow S = \left(\frac{\pi}{18} \right) (18) = \pi.$$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	$\pi/6$	$1/2$	4	2
x_2	$\pi/3$	$3/2$	2	3
x_3	$\pi/2$	2	4	8
x_4	$2\pi/3$	$3/2$	2	3
x_5	$5\pi/6$	$1/2$	4	2
x_6	π	0	1	0

$$84. (a) \text{ Each interval is } 5 \text{ min} = \frac{1}{12} \text{ hour.}$$

$$\frac{1}{24} [2.5 + 2(2.4) + 2(2.3) + \dots + 2(2.4) + 2.3] = \frac{29}{12} \approx 2.42 \text{ gal}$$

$$(b) (60 \text{ mph}) \left(\frac{12}{29} \text{ hours/gal} \right) \approx 24.83 \text{ mi/gal}$$

$$\begin{aligned} 85. y_{av} &= \frac{1}{365-0} \int_0^{365} \left[37 \sin \left(\frac{2\pi}{365} (x-101) \right) + 25 \right] dx = \frac{1}{365} \left[-37 \left(\frac{365}{2\pi} \cos \left(\frac{2\pi}{365} (x-101) \right) + 25x \right) \right]_0^{365} \\ &= \frac{1}{365} \left[\left(-37 \left(\frac{365}{2\pi} \right) \cos \left[\frac{2\pi}{365} (365-101) \right] + 25(365) \right) - \left(-37 \left(\frac{365}{2\pi} \right) \cos \left[\frac{2\pi}{365} (0-101) \right] + 25(0) \right) \right] \\ &= -\frac{37}{2\pi} \cos \left(\frac{2\pi}{365} (264) \right) + 25 + \frac{37}{2\pi} \cos \left(\frac{2\pi}{365} (-101) \right) = -\frac{37}{2\pi} \left(\cos \left(\frac{2\pi}{365} (264) \right) - \cos \left(\frac{2\pi}{365} (-101) \right) \right) + 25 \end{aligned}$$

$$\approx -\frac{37}{2\pi}(0.16705 - 0.16705) + 25 = 25^\circ\text{F}$$

$$86. \text{av}(C_v) = \frac{1}{675 - 20} \int_{20}^{675} [8.27 + 10^{-5}(26T - 1.87T^2)] dT = \frac{1}{655} \left[8.27T + \frac{13}{10^5} T^2 - \frac{0.62333}{10^5} T^3 \right]_{20}^{675}$$

$$\approx \frac{1}{655} [(5582.25 + 59.23125 - 1917.03194) - (165.4 + 0.052 - 0.04987)] \approx 5.434;$$

$$8.27 + 10^{-5}(26T - 1.87T^2) = 5.434 \Rightarrow 1.87T^2 - 26T - 283,600 = 0 \Rightarrow T \approx \frac{26 + \sqrt{676 + 4(1.87)(283,600)}}{2(1.87)}$$

$$\approx 396.45^\circ\text{C}$$

$$87. \text{Using the trapezoidal rule, } h = 15 \Rightarrow \frac{h}{2} = 7.5;$$

$$\sum mf(x_i) = 794.8 \Rightarrow \text{Area} \approx (794.8)(7.5) = 5961 \text{ ft}^2;$$

$$\text{The cost is Area} \cdot (\$2.10/\text{ft}^2) \approx (5961 \text{ ft}^2)(\$2.10/\text{ft}^2)$$

$$= \$12,518.10 \Rightarrow \text{the job cannot be done for } \$11,000.$$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	15	36	2	72
x_2	30	54	2	108
x_3	45	51	2	102
x_4	60	49.5	2	99
x_5	75	54	2	108
x_6	90	64.4	2	128.8
x_7	105	67.5	2	135
x_8	120	42	1	42

88. (a) Upper estimate:

$$3(5.30 + 5.25 + 5.04 + \dots + 1.11) = 103.05 \text{ ft}$$

Lower estimate:

$$3(5.25 + 5.04 + 4.71 + \dots + 0) = 87.15 \text{ ft}$$

$$(b) \frac{3}{2}[5.30 + 2(5.25) + 2(5.04) + \dots + 2(1.11) + 0] = 95.1 \text{ ft}$$

89. Yes. The function f , being differentiable on $[a, b]$, is then continuous on $[a, b]$. The Fundamental Theorem of Calculus says that every continuous function on $[a, b]$ is the derivative of a function on $[a, b]$.

90. The second part of the Fundamental Theorem of Calculus states that if $F(x)$ is an antiderivative of $f(x)$ on

$[a, b]$, then $\int_a^b f(x) dx = F(b) - F(a)$. In particular, if $F(x)$ is an antiderivative of $\sqrt{1+x^4}$ on $[0, 1]$, then

$$\int_0^1 \sqrt{1+x^4} dx = F(1) - F(0).$$

$$91. y(x) = \int_5^x \frac{\sin t}{t} dt + 3$$

$$92. y' = \cos x - \frac{d}{dx} \int_{\pi}^x \cos 2t \, dt = \cos x - \cos 2x \Rightarrow y'' = -\sin x + 2 \sin 2x$$

Thus, y satisfies condition i.

$$y(\pi) = \sin \pi + \int_{\pi}^{\pi} \cos 2t \, dt + 1 = 1 \text{ and } y'(\pi) = \cos \pi - \cos 2\pi = -2$$

Thus, y satisfies condition ii.

$$93. (a) g(1) = \int_1^1 f(t) \, dt = 0$$

$$(b) g(3) = \int_1^3 f(t) \, dt = -\frac{1}{2}(2)(1) = -1$$

$$(c) g(-1) = \int_1^{-1} f(t) \, dt = -\int_{-1}^1 f(t) \, dt = -\frac{1}{4}\pi(2)^2 = -\pi$$

(d) $g'(x) = f(x)$; Since $f(x) > 0$ for $-3 < x < 1$ and $f(x) < 0$ for $1 < x < 3$, $g(x)$ has a relative maximum at $x = 1$.

$$(e) g'(-1) = f(-1) = 2$$

The equation of the tangent line is $y - (-\pi) = 2(x + 1)$ or $y = 2x + 2 - \pi$

(f) $g''(x) = f'(x)$, $f'(x) = 0$ at $x = -1$ and f' is not defined at $x = 2$. The inflection points are at $x = -1$ and $x = 2$. Note that $g''(x) = f'(x)$ is undefined at $x = 1$ as well, but since $g''(x) = f'(x)$ is negative on both sides of $x = 1$, $x = 1$ is not an inflection point.

(g) Note that the absolute maximum is $g(1) = 0$ and the absolute minimum is

$$g(-3) = \int_1^{-3} f(t) \, dt = -\int_{-3}^1 f(t) \, dt = -\frac{1}{2}\pi(2)^2 = -2\pi.$$

The range of g is $[-2\pi, 0]$.

94. (a) Before the chute opens for A, $a = -32 \text{ ft/sec}^2$. Since the helicopter is hovering, $v_0 = 0 \text{ ft/sec}$

$$\Rightarrow v = \int -32 \, dt = -32t + v_0 = -32t. \text{ Then } s_0 = 6400 \text{ ft} \Rightarrow s = \int -32t \, dt = -16t^2 + s_0 = -16t^2 + 6400.$$

At $t = 4 \text{ sec}$, $s = -16(4)^2 + 6400 = 6144 \text{ ft}$ when A's chute opens;

$$(b) \text{ For B, } s_0 = 7000 \text{ ft, } v_0 = 0, a = -32 \text{ ft/sec}^2 \Rightarrow v = \int -32 \, dt = -32t + v_0 = -32t \Rightarrow s = \int -32t \, dt \\ = -16t^2 + s_0 = -16t^2 + 7000. \text{ At } t = 13 \text{ sec, } s = -16(13)^2 + 7000 = 4296 \text{ ft when B's chute opens;}$$

(c) After the chutes open, $v = -16 \text{ ft/sec} \Rightarrow s = \int -16 \, dt = -16t + s_0$. For A, $s_0 = 6144 \text{ ft}$ and for B, $s_0 = 4296 \text{ ft}$. Therefore, for A, $s = -16t + 6144$ and for B, $s = -16t + 4296$. When they hit the ground,

$$s = 0 \Rightarrow \text{for A, } 0 = -16t + 6144 \Rightarrow t = \frac{6144}{16} = 384 \text{ seconds, and for B, } 0 = -16t + 4296 \Rightarrow t = \frac{4296}{16}$$

= 268.5 seconds to hit the ground after the chutes open. Since B's chute opens 54 seconds after A's opens $\Rightarrow$ B hits the ground first.

$$\begin{aligned} 95. \text{ av}(I) &= \frac{1}{30} \int_0^{30} (1200 - 40t) dt = \frac{1}{30} [1200t - 20t^2]_0^{30} = \frac{1}{30} [(1200(30) - 20(30)^2) - (1200(0) - 20(0)^2)] \\ &= \frac{1}{30} (18,000) = 600; \text{ Average Daily Holding Cost} = (600)(\$0.03) = \$18 \end{aligned}$$

$$\begin{aligned} 96. \text{ av}(I) &= \frac{1}{14} \int_0^{14} (600 + 600t) dt = \frac{1}{14} [600t + 300t^2]_0^{14} = \frac{1}{14} [600(14) + 300(14)^2 - 0] = 4800; \text{ Average Daily} \\ \text{Holding Cost} &= (4800)(\$0.04) = \$192 \end{aligned}$$

$$\begin{aligned} 97. \text{ av}(I) &= \frac{1}{30} \int_0^{30} \left(450 - \frac{t^2}{2}\right) dt = \frac{1}{30} \left[450t - \frac{t^3}{6}\right]_0^{30} = \frac{1}{30} \left[450(30) - \frac{30^3}{6} - 0\right] = 300; \text{ Average Daily Holding Cost} \\ &= (300)(\$0.02) = \$6 \end{aligned}$$

$$\begin{aligned} 98. \text{ av}(I) &= \frac{1}{60} \int_0^{60} (600 - 20\sqrt{15}t) dt = \frac{1}{60} \int_0^{60} \left(600 - 20\sqrt{15}t^{1/2}\right) dt = \frac{1}{60} \left[600t - 20\sqrt{15}\left(\frac{2}{3}\right)t^{3/2}\right]_0^{60} \\ &= \frac{1}{60} \left[600(60) - \frac{40\sqrt{15}}{3}(60)^{3/2} - 0\right] = \frac{1}{60} \left(36,000 - \left(\frac{320}{3}\right)15^2\right) = 200; \text{ Average Daily Holding Cost} \\ &= (200)(\$0.005) = \$1.00 \end{aligned}$$

CHAPTER 4 ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS

$$1. (a) \text{ Yes, because } \int_0^1 f(x) dx = \frac{1}{7} \int_0^1 7f(x) dx = \frac{1}{7}(7) = 1$$

$$\begin{aligned} (b) \text{ No. For example, } \int_0^1 8x dx &= [4x^2]_0^1 = 4, \text{ but } \int_0^1 \sqrt{8x} dx = \left[2\sqrt{2}\left(\frac{x^{3/2}}{3/2}\right)\right]_0^1 = \frac{4\sqrt{2}}{3}(1^{3/2} - 0^{3/2}) \\ &= \frac{4\sqrt{2}}{3} \neq \sqrt{4} \end{aligned}$$

$$2. (a) \text{ True: } \int_5^2 f(x) dx = - \int_2^5 f(x) dx = -3$$

$$\begin{aligned} (b) \text{ True: } \int_{-2}^5 [f(x) + g(x)] dx &= \int_{-2}^5 f(x) dx + \int_{-2}^5 g(x) dx = \int_{-2}^2 f(x) dx + \int_2^5 f(x) dx + \int_{-2}^5 g(x) dx \\ &= 4 + 3 + 2 = 9 \end{aligned}$$

$$(c) \text{ False: } \int_{-2}^5 f(x) \, dx = 4 + 3 = 7 > 2 = \int_{-2}^5 g(x) \, dx \Rightarrow \int_{-2}^5 [f(x) - g(x)] \, dx > 0 \Rightarrow \int_{-2}^5 [g(x) - f(x)] \, dx < 0.$$

On the other hand, $f(x) \leq g(x) \Rightarrow [g(x) - f(x)] \geq 0 \Rightarrow \int_{-2}^5 [g(x) - f(x)] \, dx \geq 0$ which would be a contradiction.

$$\begin{aligned} 3. \quad y &= \frac{1}{a} \int_0^x f(t) \sin a(x-t) \, dt = \frac{1}{a} \int_0^x f(t) \sin ax \cos at \, dt - \frac{1}{a} \int_0^x f(t) \cos ax \sin at \, dt \\ &= \frac{\sin ax}{a} \int_0^x f(t) \cos at \, dt - \frac{\cos ax}{a} \int_0^x f(t) \sin at \, dt \Rightarrow \frac{dy}{dx} = \cos ax \int_0^x f(t) \cos at \, dt \\ &\quad + \frac{\sin ax}{a} \left(\frac{d}{dx} \int_0^x f(t) \cos at \, dt \right) + \sin ax \int_0^x f(t) \sin at \, dt - \frac{\cos ax}{a} \left(\frac{d}{dx} \int_0^x f(t) \sin at \, dt \right) \\ &= \cos ax \int_0^x f(t) \cos at \, dt + \frac{\sin ax}{a} (f(x) \cos ax) + \sin ax \int_0^x f(t) \sin at \, dt - \frac{\cos ax}{a} (f(x) \sin ax) \\ &\Rightarrow \frac{dy}{dx} = \cos ax \int_0^x f(t) \cos at \, dt + \sin ax \int_0^x f(t) \sin at \, dt. \text{ Next,} \\ \frac{d^2y}{dx^2} &= -a \sin ax \int_0^x f(t) \cos at \, dt + (\cos ax) \left(\frac{d}{dx} \int_0^x f(t) \cos at \, dt \right) + a \cos ax \int_0^x f(t) \sin at \, dt \\ &\quad + (\sin ax) \left(\frac{d}{dx} \int_0^x f(t) \sin at \, dt \right) = -a \sin ax \int_0^x f(t) \cos at \, dt + (\cos ax) f(x) \cos ax \\ &\quad + a \cos ax \int_0^x f(t) \sin at \, dt + (\sin ax) f(x) \sin ax = -a \sin ax \int_0^x f(t) \cos at \, dt + a \cos ax \int_0^x f(t) \sin at \, dt + f(x). \\ \text{Therefore, } y'' + a^2 y &= a \cos ax \int_0^x f(t) \sin at \, dt - a \sin ax \int_0^x f(t) \cos at \, dt + f(x) \\ &\quad + a^2 \left(\frac{\sin ax}{a} \int_0^x f(t) \cos at \, dt - \frac{\cos ax}{a} \int_0^x f(t) \sin at \, dt \right) = f(x). \text{ Note also that } y'(0) = y(0) = 0. \end{aligned}$$

$$\begin{aligned} 4. \quad x &= \int_0^y \frac{1}{\sqrt{1+4t^2}} \, dt \Rightarrow \frac{d}{dx}(x) = \frac{d}{dx} \int_0^y \frac{1}{\sqrt{1+4t^2}} \, dt = \frac{d}{dy} \left[\int_0^y \frac{1}{\sqrt{1+4t^2}} \, dt \right] \left(\frac{dy}{dx} \right) \text{ from the chain rule} \\ &\Rightarrow 1 = \frac{1}{\sqrt{1+4y^2}} \left(\frac{dy}{dx} \right) \Rightarrow \frac{dy}{dx} = \sqrt{1+4y^2}. \text{ Then } \frac{d^2y}{dx^2} = \frac{d}{dx}(\sqrt{1+4y^2}) = \frac{d}{dy}(\sqrt{1+4y^2}) \left(\frac{dy}{dx} \right) \end{aligned}$$

$$= \frac{1}{2}(1+4y^2)^{-1/2} (8y) \left(\frac{dy}{dx} \right) = \frac{4y \left(\frac{dy}{dx} \right)}{\sqrt{1+4y^2}} = \frac{4y(\sqrt{1+4y^2})}{\sqrt{1+4y^2}} = 4y. \text{ Thus } \frac{d^2y}{dx^2} = 4y, \text{ and the constant of}$$

proportionality is 4.

$$5. (a) \int_0^{x^2} f(t) dt = x \cos \pi x \Rightarrow \frac{d}{dx} \int_0^{x^2} f(t) dt = \cos \pi x - \pi x \sin \pi x \Rightarrow f(x^2)(2x) = \cos \pi x - \pi x \sin \pi x$$

$$\Rightarrow f(x^2) = \frac{\cos \pi x - \pi x \sin \pi x}{2x}. \text{ Thus, } x = 2 \Rightarrow f(4) = \frac{\cos 2\pi - 2\pi \sin 2\pi}{4} = \frac{1}{4}$$

$$(b) \int_0^{f(x)} t^2 dt = \left[\frac{t^3}{3} \right]_0^{f(x)} = \frac{1}{3}(f(x))^3 \Rightarrow \frac{1}{3}(f(x))^3 = x \cos \pi x \Rightarrow (f(x))^3 = 3x \cos \pi x \Rightarrow f(x) = \sqrt[3]{3x \cos \pi x}$$

$$\Rightarrow f(4) = \sqrt[3]{3(4) \cos 4\pi} = \sqrt[3]{12}$$

$$6. \int_0^a f(x) dx = \frac{a^2}{2} + \frac{a}{2} \sin a + \frac{\pi}{2} \cos a \text{ and let } F(a) = \int_0^a f(t) dt \Rightarrow f(a) = F'(a). \text{ Now } F(a) = \frac{a^2}{2} + \frac{a}{2} \sin a + \frac{\pi}{2} \cos a$$

$$\Rightarrow f(a) = F'(a) = a + \frac{1}{2} \sin a + \frac{a}{2} \cos a - \frac{\pi}{2} \sin a \Rightarrow f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{1}{2} \sin \frac{\pi}{2} + \frac{\left(\frac{\pi}{2}\right)}{2} \cos \frac{\pi}{2} - \frac{\pi}{2} \sin \frac{\pi}{2} = \frac{\pi}{2} + \frac{1}{2} - \frac{\pi}{2} = \frac{1}{2}$$

$$7. \int_1^b f(x) dx = \sqrt{b^2+1} - \sqrt{2} \Rightarrow f(b) = \frac{d}{db} \int_1^b f(x) dx = \frac{1}{2}(b^2+1)^{-1/2}(2b) = \frac{b}{\sqrt{b^2+1}} \Rightarrow f(x) = \frac{x}{\sqrt{x^2+1}}$$

$$8. \text{ The derivative of the left side of the equation is: } \frac{d}{dx} \left[\int_0^x \left[\int_0^u f(t) dt \right] du \right] = \int_0^x f(t) dt; \text{ the derivative of the right}$$

$$\text{side of the equation is: } \frac{d}{dx} \left[\int_0^x f(u)(x-u) du \right] = \frac{d}{dx} \int_0^x f(u) x du - \frac{d}{dx} \int_0^x u f(u) du$$

$$= \frac{d}{dx} \left[x \int_0^x f(u) du \right] - \frac{d}{dx} \int_0^x u f(u) du = \int_0^x f(u) du + x \left[\frac{d}{dx} \int_0^x f(u) du \right] - x f(x) = \int_0^x f(u) du + x f(x) - x f(x)$$

$$= \int_0^x f(u) du. \text{ Since each side has the same derivative, they differ by a constant, and since both sides equal 0}$$

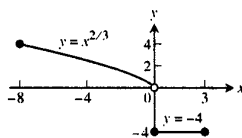
$$\text{when } x = 0, \text{ the constant must be 0. Therefore, } \int_0^x \left[\int_0^u f(t) dt \right] du = \int_0^x f(u)(x-u) du.$$

$$9. \frac{dy}{dx} = 3x^2 + 2 \Rightarrow y = \int (3x^2 + 2) dx = x^3 + 2x + C. \text{ Then } (1, -1) \text{ on the curve } \Rightarrow 1^3 + 2(1) + C = -1 \Rightarrow C = -4$$

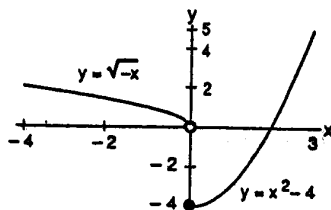
$$\Rightarrow y = x^3 + 2x - 4$$

10. The acceleration due to gravity downward is $-32 \text{ ft/sec}^2 \Rightarrow v = \int -32 \, dt = -32t + v_0$, where v_0 is the initial velocity $\Rightarrow v = -32t + 32 \Rightarrow s = \int (-32t + 32) \, dt = -16t^2 + 32t + C$. If the release point is $s = 0$, then $C = 0 \Rightarrow s = -16t^2 + 32t$. Then $s = 17 \Rightarrow 17 = -16t^2 + 32t \Rightarrow 16t^2 - 32t + 17 = 0$. The discriminant of this quadratic equation is -64 which says there is no real time when $s = 17 \text{ ft}$. You had better duck.

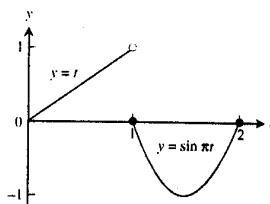
$$\begin{aligned}
 11. \quad \int_{-8}^3 f(x) \, dx &= \int_{-8}^0 x^{2/3} \, dx + \int_0^3 -4 \, dx \\
 &= \left[\frac{3}{5} x^{5/3} \right]_{-8}^0 + [-4x]_0^3 \\
 &= \left(0 - \frac{3}{5} (-8)^{5/3} \right) + (-4(3) - 0) = \frac{96}{5} - 12 \\
 &= \frac{36}{5}
 \end{aligned}$$



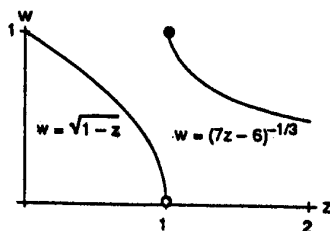
$$\begin{aligned}
 12. \quad \int_{-4}^3 f(x) \, dx &= \int_{-4}^0 \sqrt{-x} \, dx + \int_0^3 (x^2 - 4) \, dx \\
 &= \left[-\frac{2}{3} (-x)^{3/2} \right]_{-4}^0 + \left[\frac{x^3}{3} - 4x \right]_0^3 \\
 &= \left[0 - \left(-\frac{2}{3} (4)^{3/2} \right) \right] + \left(\frac{3^3}{3} - 4(3) \right) - 0 \\
 &= \frac{16}{3} - 3 = \frac{7}{3}
 \end{aligned}$$



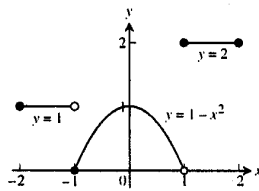
$$\begin{aligned}
 13. \quad \int_0^2 f(t) \, dt &= \int_0^1 t \, dt + \int_1^2 \sin \pi t \, dt \\
 &= \left[\frac{t^2}{2} \right]_0^1 + \left[-\frac{1}{\pi} \cos \pi t \right]_1^2 \\
 &= \left(\frac{1}{2} - 0 \right) + \left[-\frac{1}{\pi} \cos 2\pi - \left(-\frac{1}{\pi} \cos \pi \right) \right] \\
 &= \frac{1}{2} - \frac{2}{\pi}
 \end{aligned}$$



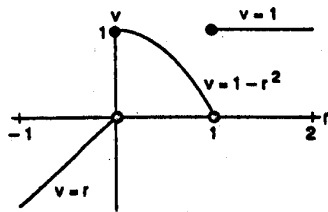
$$\begin{aligned}
 14. \quad \int_0^2 f(z) \, dz &= \int_0^1 \sqrt{1-z} \, dz + \int_1^2 (7z-6)^{-1/3} \, dz \\
 &= \left[-\frac{2}{3} (1-z)^{3/2} \right]_0^1 + \left[\frac{3}{14} (7z-6)^{2/3} \right]_1^2 \\
 &= \left[-\frac{2}{3} (1-1)^{3/2} - \left(-\frac{2}{3} (1-0)^{3/2} \right) \right] \\
 &\quad + \left[\frac{3}{14} (7(2)-6)^{2/3} - \frac{3}{14} (7(1)-6)^{2/3} \right] \\
 &= \frac{2}{3} + \left(\frac{6}{7} - \frac{3}{14} \right) = \frac{55}{42}
 \end{aligned}$$



$$\begin{aligned}
 15. \int_{-2}^2 f(x) \, dx &= \int_{-2}^{-1} dx + \int_{-1}^1 (1-x^2) \, dx + \int_1^2 2 \, dx \\
 &= [x]_{-2}^{-1} + \left[x - \frac{x^3}{3} \right]_{-1}^1 + [2x]_1^2 \\
 &= -1 - (-2) + \left(1 - \frac{1^3}{3} \right) - \left(-1 - \frac{(-1)^3}{3} \right) + 2(2) - 2(1) \\
 &= 1 + \frac{2}{3} - \left(-\frac{2}{3} \right) + 4 - 2 = \frac{13}{3}
 \end{aligned}$$



$$\begin{aligned}
 16. \int_{-1}^2 h(r) \, dr &= \int_{-1}^0 r \, dr + \int_0^1 (1-r^2) \, dr + \int_1^2 dr \\
 &= \left[\frac{r^2}{2} \right]_{-1}^0 + \left[r - \frac{r^3}{3} \right]_0^1 + [r]_1^2 \\
 &= 0 - \frac{(-1)^2}{2} + \left(1 - \frac{1^3}{3} \right) - 0 + 2 - 1 \\
 &= -\frac{1}{2} + \frac{2}{3} + 1 = \frac{7}{6}
 \end{aligned}$$



$$\begin{aligned}
 17. \text{Ave. value} &= \frac{1}{b-a} \int_a^b f(x) \, dx = \frac{1}{2-0} \int_0^2 f(x) \, dx = \frac{1}{2} \left[\int_0^1 x \, dx + \int_1^2 (x-1) \, dx \right] \\
 &= \frac{1}{2} \left[\frac{x^2}{2} \right]_0^1 + \frac{1}{2} \left[\frac{x^2}{2} - x \right]_1^2 \\
 &= \frac{1}{2} \left[\frac{1^2}{2} - 0 + \left(\frac{2^2}{2} - 2 \right) - \left(\frac{1^2}{2} - 1 \right) \right] = \frac{1}{2}
 \end{aligned}$$

$$18. \text{Ave. value} = \frac{1}{b-a} \int_a^b f(x) \, dx = \frac{1}{3-0} \int_0^3 f(x) \, dx = \frac{1}{3} \left[\int_0^1 dx + \int_1^2 0 \, dx + \int_2^3 dx \right] = \frac{1}{3} [1 - 0 + 0 + 3 - 2] = \frac{2}{3}$$

$$19. f(x) = \int_{1/x}^x \frac{1}{t} \, dt \Rightarrow f'(x) = \frac{1}{x} \left(\frac{dx}{dx} \right) - \left(\frac{1}{x} \right) \left(\frac{d}{dx} \left(\frac{1}{x} \right) \right) = \frac{1}{x} - x \left(-\frac{1}{x^2} \right) = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$$

$$\begin{aligned}
 20. f(x) &= \int_{\cos x}^{\sin x} \frac{1}{1-t^2} \, dt \Rightarrow f'(x) = \left(\frac{1}{1-\sin^2 x} \right) \left(\frac{d}{dx} (\sin x) \right) - \left(\frac{1}{1-\cos^2 x} \right) \left(\frac{d}{dx} (\cos x) \right) \\
 &= \frac{\cos x}{\cos^2 x} + \frac{\sin x}{\sin^2 x} \\
 &= \frac{1}{\cos x} + \frac{1}{\sin x}
 \end{aligned}$$

$$21. g(y) = \int_{\sqrt{y}}^{2\sqrt{y}} \sin t^2 \, dt \Rightarrow g'(y) = \left(\sin(2\sqrt{y})^2 \right) \left(\frac{d}{dy} (2\sqrt{y}) \right) - \left(\sin(\sqrt{y})^2 \right) \left(\frac{d}{dy} (\sqrt{y}) \right) = \frac{\sin 4y}{\sqrt{y}} - \frac{\sin y}{2\sqrt{y}}$$

22. $f(x) = \int_x^{x+3} t(5-t) \, dt \Rightarrow f'(x) = (x+3)(5-(x+3))\left(\frac{d}{dx}(x+3)\right) - x(5-x)\left(\frac{dx}{dx}\right) = (x+3)(2-x) - x(5-x)$
 $= 6 - x - x^2 - 5x + x^2 = 6 - 6x$. Thus $f'(x) = 0 \Rightarrow 6 - 6x = 0 \Rightarrow x = 1$. Also, $f''(x) = -6 < 0 \Rightarrow x = 1$ gives a maximum.
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NOTES:

TECHNOLOGY NOTES:



